

Equation of the Universe:

Atomic Ionization

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Author: Glenn R. King

Independent Researcher, Akron, Ohio, USA

Email: Gking@dms-helper.com

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Equation of the Universe — Core Theory (Rev 3.45 or later)

§0 — Introduction

0.1 Definition

Ionization is the discrete removal of electron participation from an atomic equilibrium state. Within the EOTU framework, an atom is not a proton orbited by electrons, but a coherent multi-region structure whose stable separations are set by phase-compatible proton–electron and electron–electron field relationships. Ionization therefore does not represent a particle escaping an orbit. It represents the loss of a participating electron from an allowed shell configuration, followed by re-closure of the remaining atomic structure at a new admissible equilibrium.

0.2 Conditions Required

For a one-electron atom, the total state is the coupled proton–electron corridor state, and the allowed radial ladder follows directly from the resolved closure relation. For a multi-electron atom, the total state is the nucleus together with the sum of participating electron regions, so each ionization step changes the full equilibrium configuration rather than removing energy from an otherwise unchanged atom.

Across the light elements, ionization follows shell structure. The first shell holds two electrons and the second shell holds up to eight. Electrons are removed one at a time from the active outer shell, so the ionization sequence is governed first by outer-shell depletion, then by shell closure, and finally by penetration into the inner shell. The large jumps in ionization energy therefore mark structural boundaries in the atomic configuration rather than arbitrary numerical discontinuities.

Atomic shell distances in this regime are governed by the combined proton–electron wavelength lever. The proton wavelength and electron wavelength define the admissible shell-spacing ladder, while the hydrogen ground-state separation provides the base anchor for the first ionization construction across the row. From this, the repeated ionization pattern from beryllium through neon arises as a structural consequence of the same underlying wavelength rule, with lithium acting as the bridge from first-shell closure into the second-shell sequence.

Ionization therefore has a single meaning within the framework: it is the stepwise reduction of a coherent atomic electron structure through successive allowed re-closures. Multi-electron ionization is governed by shell occupancy and structural repetition, while the terminal one-electron state reduces to the hydrogen-like limit. From this point forward, the task is not to assign empirical trends, but to derive each ionization level from the admissible geometry and closure of the atom itself.

§1 — Properties

1.1 Reference values

- Coherent Phase Packet (CPP): Fundamental oscillating unit of the EOTU lattice.
- θ is the intra-cycle phase parameter of the King recurrence (not recorded in Constellus).
- Phase Tag (Φ_{cpp}): Fixed phase offset defined at freeze-out; one of $\{0, \pi/2, \pi, 3\pi/2\}$.
- Z is the **fabric property impedance** = 376.730313412Ω
- Constellus Snapshot Ledger: Record of local CPP state changes at end of each King cycle.
- CMB spectral-distortion limits: $|\epsilon-1| \leq 1.7 \times 10^{-5}$ (μ/y era) for the Zero-Phase baseline
- Fine-structure constant stability: $\Delta\alpha/\alpha$ constrained to $\lesssim 10^{-5}$ on cosmological baselines.
- Baryonic / Dark-Matter Ratio $\sigma = \sin(0.42\pi) \approx 0.968583$
- $\lambda_k = 2.809321648 \times 10^{-20}$ m (first principle derivation)

0.1 Reference Documents

From document EOTU_CPP_Base_v3.y.z, **The 4 Primordial CPPs at eigen States** Φ_{cpp} are:

- **Zeteon** $\Phi_Z = 0$
- **Emeon** $\Phi_E = \pi/2$
- **Uniteon** $\Phi_U = \pi$,
- **Deniteon** $\Phi_D = 3\pi/2$. (revised in Version 3.41.17)

These values were derived from ionization energy and first principal values in a separate document

- Shell 1 (radius) = $S_1 = 29,447,706 L_0$ (derived)
- Shell 2 (radius) = $S_2 = 105,124,156 L_0$ (derived)
- Shell 3 (radius) = $S_3 = 121,091,309 L_0$ (derived)
- Shell 4 (radius) = $S_4 = 153,576,343 L_0$ (derived)
-
- Proton-Electron lambda = $\lambda_{pe} = \lambda_p * \lambda_e = 1836 * 816 * 1,498,176 L_0$ (derived)
- Electron-Electron lambda = $\lambda_{ee} = \lambda_e * \lambda_e = 816 * 816 = 665,856 L_0$ (derived)
- Proton-Proton lambda = $\lambda_{pp} = \lambda_p * \lambda_p = 1836 * 1836 = 3,370,896 L_0$ (derived)

0.2 Measured anchors to use (choose one primary, keep the others as checks)

Our system is based on two coherent wave sources forming a stable phase-locked separation not electron orbiting proton, whose equilibrium separation is determined by wave compatibility.

Values used or established in this section are:

- $L_0 = 64$ cells (theory fundamental)
- Ground-state ionization energy (hydrogen Measured): $E_H = 13.598$ eV
- Triad phase offset $\delta = 120^\circ = \frac{2\pi}{3}$
- $n = 16033$ (Derived in this document)
- $r_{\text{cells}} \approx 1.885 \times 10^9$ cells (Bohr Radius Derived and in Terms of cells) = 29,436,894 L_0
- Proton halo wavelength: $\lambda_p = 1836 L_0 = 1836 \times 64 = 117,504$ cells (see proton Document)
- $k_p = \frac{2\pi}{\lambda_p}$ rad/cell = $\frac{2\pi}{117,504}$ rad/cell
- Electron halo wavelength: $\lambda_e = 816 L_0 = 816 \cdot 64 = 52,224$ cells (see Elec Document)
- Electron wavenumber: $k_e = \frac{2\pi}{\lambda_e}$
- Spectral radii: $r_n = a_0 n^2$
- Spectral ladder: $E_n = -\frac{13.598 \text{ eV}}{n^2}$

Use these as factual targets:

- Bohr radius: $a_0 = 5.29177210903 \times 10^{-11}$ m $\approx 1.884 \times 10^9$ lattice cells
- Ground-state ionization energy: $E_H = 13.5984$ eV
- 21-cm hyperfine transition: $f_{hf} \approx 1.42040575177$ GHz

0.2.1 Editorial note on spreadsheet

This table represents a one-electron atom (protium) modeled as two phase-coupled regions. The ground state is purely radial. At the first excited level (n=2), two equilibrium types exist:

- symmetric (2s)
- directional (2p)

The directional state is represented in three axis-defined orientations:

- axial (z)
- transverse (x)
- transverse (y)

These three rows represent the **same energy state**, not different energies.

0.2.2 single electron atom

We have already resolved the radial ladder and the paired closure-class diagnostics for one electron.

$$k_r r + k_\theta \theta + k_\phi \phi + \delta = (2n + 1)\pi$$

The corridor supports spherical phase propagation, and for the resolved single-electron case the angular terms are inactive:

$$k_\theta \theta = 0, k_\phi \phi = 0$$

so the closure reduces to (equivalent spherical states ("s"))

$$k_r r + \delta = (2n + 1)\pi$$

with the total corridor state is for a single electron atom:

$$\Psi_{tot} = \Psi_p + \Psi_e$$

0.2.3 Multi electron atom

polar closure rule, azimuthal closure rule, node/lobe geometry derived from r, θ, ϕ directly

$$k_r r + k_\theta \theta + k_\phi \phi + \delta = ???$$

Each atomic state corresponds to a **unique equilibrium field configuration** that produces a unique scattering signature.

If k_θ or k_ϕ are non-zero, the equilibrium state defines a preferred phase structure in angle, and different orientations relative to the beam can produce different scattering outcomes.

The total corridor state is for a multi nuclei or multi electron atom:

$$\Psi_{tot} = \Psi_N + \Sigma \Psi_{e,i}$$

0.3 for all elements Z=1...10

- The elements from hydrogen through neon are governed by a two-shell electron structure.
- The first shell holds 2 electrons.
- The second shell holds up to 8 electrons.
- Electrons are removed one at a time, beginning with the outer shell.
- For Z = 1...10, ionization proceeds by outer-shell depletion until only the inner-shell structure remains.
- Neon is the closure of the second shell.

0.3.1 Shell-distance rule

Atomic shell distances in this range are governed by the combined proton–electron wavelength lever.

The proton wavelength is

$$\lambda_p = 1836 L_0$$

The electron wavelength is

$$\lambda_e = 816 L_0$$

The combined wavelength unit is

$$\lambda_{pe} = \lambda_p \lambda_e = 1,498,176 L_0$$

Allowed shell distances are expressed as a base distance plus or minus an integer multiple of the combined wavelength unit.

0.3.2 Ground-state anchor

- The hydrogen ground-state anchor is

$$H_{GS} = 29,447,705.851 L_0$$

- Deuterium and tritium use the same hydrogen anchor with a neutron correction term.
- This hydrogen anchor provides the base reference for the first ionization construction across the row.

0.3.3 First-ionization construction

For most elements in the range Z = 2...10, the first-ionization shell distance is constructed from

$$2(\text{Ground State}) - \frac{1}{2}H_{GS} \pm n\lambda_{pe}$$

where $n \in \mathbb{Z}$.

In expanded form, this becomes

$$44,171,559 L_0 \pm n(1,498,176 L_0)$$

Lithium is the bridge exception in this row and uses

$$3(\text{Ground State}) - \frac{1}{2}H_{GS} \pm n\lambda_{pe}$$

Lithium is therefore treated as a special transition case rather than as the standard row rule.

0.3.4 Ionization-order rule

- Successive ionization energies are generated by removing electrons one at a time from the active outer shell.
- The second-shell electrons do not follow a purely hydrogen-like n^2 ladder while the atom remains a multi-electron system.
- Near shell closure, the ionization pattern reflects shell structure and closure behavior rather than a one-electron hydrogenic law.
- Fluorine and neon confirm this closure behavior: the second shell closes at neon, and the major jump occurs when ionization begins to penetrate the inner shell.

0.3.5 Final one-electron rule

The final one-electron ionization state is hydrogen-like throughout $Z = 1 \dots 10$. The terminal one-electron state is calculated from

$$\frac{H_{GS}}{(Z-1)^2}$$

when the ion has been reduced to a one-electron system.

This rule applies only to the true one-electron end state, not to earlier multi-electron stripped states.

0.3.6 Structural interpretation

- Hydrogen and helium establish the first-shell anchor behavior.
- Lithium is the bridge from the helium closure into the second-shell sequence.
- Beryllium through neon largely follow the same repeatable lever structure.
- The second shell closes at neon.
- The final stripped state is governed by the hydrogen-like one-electron rule.

0.3 Electron Reach $R_{\text{eff},e} N_{E+ZZZ} = 4$

$$k_{\text{CPP}} (UUD + ZZZ) = 4^2 = 16$$

$$R_{\text{eff}}(k) = (51 k_{\text{CPP}}) L_0 = 51(16) L_0 = 816 L_0$$

The composite core for baryonic then becomes $k_{\text{CPP}} (E + ZZZ) = 4^2 = 16$:

$$R_{\text{eff, baryonic}} = \frac{8}{51} 816 L_0 = 128 L_0$$

Electron halo wavelength: $\lambda_e = 816 L_0$

0.4 Proton Reach $R_{\text{eff},p} N_{UUD+ZZZ} = 6$

$$k_{\text{CPP}} (UUD + ZZZ) = 6^2 = 36$$

$$R_{\text{eff}}(k) = (51 k_{\text{CPP}}) L_0 = 51(36) L_0 = 1836 L_0$$

The composite core for baryonic then becomes:

$$R_{\text{eff, baryonic}} = \frac{8}{51} 1836 L_0 = 288 L_0$$

Proton halo wavelength: $\lambda_p = 1836 L_0$

0.5 Neutron Reach $R_{\text{eff},n} N_{UUDZZZ+EZZZ} = 10$

$$k_n = 10^2 = 100$$

$$R_{\text{eff}}(k) = (51 k_n) L_0, \quad R_{\text{eff},n} = 51(100) L_0 = 5100 L_0$$

The composite core for baryonic then becomes:

$$R_{\text{eff, baryonic}} = \frac{8}{51} 5100 L_0 = 800 L_0$$

$$\lambda_{pe} = 1,498,176 L_0, \quad \lambda_{pp} = 3,370,896 L_0, \quad \frac{\lambda_p}{\lambda_e} = 2.25$$

0.6 Electron Shell Law and Active Radius

Electron occupy discrete 3D curvature shells labeled by an integer index $n = 1, 2, 3, \dots$ Shell n sits at absolute layer index

$$L = L_{\text{core}}(Z) + n,$$

so its radius is

$$r_n(Z) = (L_{\text{core}}(Z) + n) R_p.$$

The cumulative electron capacity of shells up to and including shell n is defined by

$$E_{\text{max}}(n) = 2n^2,$$

so that the capacity of shell n alone is

$$\Delta E_n = E_{\text{max}}(n) - E_{\text{max}}(n - 1) = 2n^2 - 2(n - 1)^2 = 4n - 2.$$

For a neutral atom with charge Z , the total Electron count is $n_e = Z$. The number of fully filled electron shells and the outermost occupied shell index are then

$$n_{\text{full}}(Z) = \sqrt{\frac{Z}{2}}, \quad n_{\text{out}}(Z) = \sqrt{\frac{Z}{2}}.$$

The electron envelope radius is

$$r_{\text{Electron}}(Z) = (L_{\text{core}}(Z) + n_{\text{out}}(Z)) R_p.$$

The total active atomic radius is taken as

$$r_{\text{atomic}}(Z) = \max (R_{\text{core}}(Z), R_{\text{Electron}}(Z)),$$

§2 – Protium The first Atomic Structure

The hydrogen atom is a two-source wave system whose equilibrium separation is determined by phase closure between a proton field and an electron field, producing the correct atomic radius, energy ladder, and spectral transitions without invoking orbital mechanics.

Protium is a two-Region steady-state system consisting of one proton Region and one electron Region.

The hydrogen ground-state separation is represented in the current formulation by the locked closure relation:

$$r = \left(n + \frac{1}{6}\right) \lambda_p$$

2.1 Phase-Closure Condition

Solving for r based on proton wavelength

$$r_p = \frac{(2n + 1)\pi - \delta}{k_p}$$

Substitute $k_p = 2\pi/\lambda_p$:

$$r_p = ((2n + 1)\pi - \delta) \frac{\lambda_p}{2\pi}$$

Now substitute $\delta = \frac{2\pi}{3}$:

$$r = \left((2n + 1)\pi - \frac{2\pi}{3} \right) \frac{\lambda_p}{2\pi} = \left((2n + 1) - \frac{2}{3} \right) \frac{\lambda_p}{2} = \left(2n + \frac{1}{3} \right) \frac{\lambda_p}{2}$$

$$\boxed{r_p = \left(n + \frac{1}{6} \right) \lambda_p}$$

The fractional term $\frac{1}{6} \lambda_p$ arises directly from the 120° triad phase offset when substituted into the closure

For $n = 16038$, $\lambda_p = 1836 L_0 = 117,504$ cells

$$r = \left(16038 + \frac{1}{6} \right) 117,504 = 16038 \cdot 117,504 + \frac{1}{6} \cdot 117,504$$

$$r = 1,884,633,600 + 19,584 = 1,884,653,184 \text{ cells}$$

$$\boxed{r_{\text{cells}} = 1,884,653,184 \times 10^9 \text{ cells} = 29,447,706 L_0}$$

Observed: $a_0 = 1,884,653,184 \times 10^9$ cells

The hydrogen separation scale is corridor phase closure using the proton halo wavelength and a 120° structural phase offset. A separation at which corridor phase closure remains self-consistent under small radial perturbations defines a stable equilibrium separation. The difference between the triad phase fraction and the baryonic fraction reduces to

$$\frac{1}{6} - \frac{8}{51} = \frac{1}{102}$$

The value **102** corresponds exactly to the halo diameter of a CPP since the halo radius is 51 L_0 .

$$\text{Constant} = \frac{Z_c e^2}{8\pi \lambda_k} = 4.106111439 \times 10^{-9}$$

$$C_{L_0} = \frac{C_{\text{cell}}}{64}$$

$$C_{L_0} = 6.415799123 \times 10^{-11} \text{ J} = 4.0044268446 \times 10^8 \text{ eV}$$

$$E(\text{eV}) = \frac{4.0044268446 \times 10^8}{r_{L_0}}, \quad E(\text{MeV}) = \frac{400.44268446}{r_{L_0}}$$

$$E_H = -\frac{Zc}{8\pi r_{\text{cells,H}}} \frac{e^2}{\lambda_k} \approx -13.598434 \text{ eV.}$$

where $r_{\text{cells,H}} = 29,447,705.851 L_0 \approx 29,447,706 L_0$ (protium)

$$E_D = -\frac{Zc}{8\pi r_{\text{cells,D}}} \frac{e^2}{\lambda_k} \approx -13.602134123 \text{ eV.}$$

where $r_{\text{cells,D}} = 29,439,695.334 L_0 \approx 29,439,695 L_0$ (Deuteron)

$$E_T = -\frac{Zc}{8\pi r_{\text{cells,T}}} \frac{e^2}{\lambda_k} \approx -13.603365214 \text{ eV.}$$

where $r_{\text{cells,T}} = 29,437,031.069 L_0 \approx 29,437,031 L_0$ (Tritium)

$$\Delta r_D = r_D - r_H = -8,011 L_0$$

$$\Delta r_T = r_T - r_H = -10,675 L_0$$

Measured Value

$$E_{\text{ion}}(\text{H}) = 13.598434599702 \text{ eV}$$

2.2 Bohr Radius Proton-Electron separation at ground state

Protium separation is an emergent equilibrium of two steady CPP-region sources coupled with the lattice Green-function response of the dormant fabric the relevant length scale is the halo wavelength and its associated phase-propagation constant. The Green response is phase-propagating (Helmholtz-type), not static.

$$r_{\text{pred}} = \left(n + \frac{1}{6}\right) \lambda_p = 1,884,653,184 \times 10^9 \text{ cells}$$

Cross-check (length): Compare r_{pred} to the measured Bohr radius expressed in cells. Agreement here validates that the energy-anchored separation and the phase-closure separation are mutually consistent at the atomic scale.

Bohr radius Measured

$$a_0 = 5.29177210903 \times 10^{-11} \text{ meters}$$

Convert Bohr radius to lattice cells

$$\text{Bohr radius} \equiv N_{a_0} = \frac{a_0}{\lambda_k} = \frac{5.29177210903 \times 10^{-11}}{2.809321648 \times 10^{-20}} \approx 1.883963257011 \times 10^9 \text{ lattice Cells}$$

Percent Error:

$$\Delta r = 1,884,653,184 - 1,883,963,257 = 689,927 \text{ cells}$$

$$\% \text{ error} = \frac{\Delta r}{\text{Measured}} = \frac{689,927}{1.883963257011} \times 100 \approx 0.03662\%$$

Other Radius:

- $r_{\text{He}^+} = 2.645886054515 \times 10^{-11} \text{ m.} \approx 941,823,823 \text{ cells} \approx 14,715,997$
- $r_{\text{Li}^{2+}} = 1.7639240363433332 \times 10^{-11} \text{ m.}$
- $r_{\text{Be}^{3+}} = 1.3229430272575 \times 10^{-11} \text{ m}$
-

2.3 Mass of the p-n core

$$m_{pn}^* = \sqrt{\frac{N_o}{3}}$$

curvature-only Mass from radius 2D circular inventory 2D circular (disk) reach inventory: $N_o = \pi r^2$

$$N_o(k_{pn}) = \pi(r k_{pn})^2$$

Where

- $N_{pn} = 16 (2p + e) = (12 + 4) = 16$
- $k_{CPP}(UUD) \equiv k_{pn} = (N_{pn})^2 = (16)^2 = 256$
- $r = 1024 L_0$

1. Square the radius term

$$r^2 = (1024)^2 = 1\,048\,576$$

2. Multiply by π

$$N_o = \pi \times 1\,048\,576 \approx 3.14159265 \times 1\,048\,576 \approx 3\,294\,198$$

3. Divide by 3

$$\frac{N_o}{3} = \frac{3\,294\,198}{3} \approx 1\,098\,066$$

4. Take the square root

$$m_{pn}^* = \sqrt{1\,098\,066} \approx 1047.89$$

1.5 p-n Energy

$$E_R = \Gamma(m_{UUD} + m_{Z\Delta} + \Delta m_{int})^2 = \Gamma(1047.89)^2 \approx 1.10007 \text{ MeV}$$

2.4 - Protium energy

So the **curvature** must be built from the **cycle-averaged coupling term**

$$\mu_{int}(r) \propto \langle \Psi_p(r, t) \Psi_e(r, t) \rangle_{\text{cycle}}$$

Cycle-average of a product of cosines gives a cosine of the **relative phase**:

$$\langle \Psi_p \Psi_e \rangle \propto \frac{A_p A_e}{r^2} \cos(\Delta\Phi(r))$$

where

$$\Delta\Phi(r) = (k_p - k_e)r + (\Phi_p - \Phi_e) \text{ (plus your fixed closure offsets like } \delta = 120^\circ \text{)}$$

This is the first-principles bridge: **energy comes from coupling** of the two steady corridor sources.

$$E_H(S) = \Gamma [\mu_{int}(S)]^2$$

2.4.1 Protium energy Using geometry

$$E_H = -\frac{Zc}{8\pi} \frac{e^2}{r_{\text{cells,H}} \lambda_k} \approx -13.598434 \text{ eV.}$$

where $r = 29,447,706 L_0 = 1,884,653,184 \times 10^9$ cells

2.4.2 Higher Energy State

See photon documentation for details

$$k_r r + k_\theta \theta + k_\phi \phi$$

Two types of equilibrium, one of which has multiple orientations

A photon, in the same language, is an **incoming corridor mode** with its own $(k_\gamma, \omega_\gamma)$ that *adds* to the local corridor state:

$$\Psi_{\text{tot}} = \Psi_p + \Psi_e + \Psi_\gamma$$

Coupling (absorption / emission / scattering) occurs when adding Ψ_γ produces a **stable, allowed re-closure** of the proton–electron corridor boundary condition (i.e., the coupled system can settle into a new coherent configuration rather than “rejecting” the mode as a transient).

These are the measured anchors you already cite as targets in the protium section:

- **Lyman- α** (a dominant hydrogen UV line) tabulated by NIST around 1215.67 Å (121.567 nm).
- **21-cm hyperfine line** at ~1420.4 MHz (radio).

EOTU prediction target here: the p–e corridor has allowed standing/eigenmodes determined by separation + boundary compatibility, so transitions correspond to discrete allowed changes in the corridor mode structure that emit/absorb photons at the measured frequencies.

2.4.3 Energy Ladder for Single Electron

The hydrogen document already locks the spectral radius and energy ladder as

$$r_n = a_0 n^2, \quad E_n = -\frac{13.598 \text{ eV}}{n^2}$$

with the ground-state radius in cells given by the solved closure relation.

Here are the first few states:

$$E_1 = -13.598 \text{ eV}, \quad E_2 = -\frac{13.598}{4} = -3.3995 \text{ eV},$$

$$E_3 = -\frac{13.598}{9} = -1.5109 \text{ eV}, \quad E_4 = -\frac{13.598}{16} = -0.8499 \text{ eV}$$

For the first major line, $n = 2 \rightarrow 1$,

$$\Delta E_{21} = 13.598 \left(1 - \frac{1}{4}\right) = 10.1985 \text{ eV}.$$

For $n = 3 \rightarrow 2$,

$$\Delta E_{32} = 13.598 \left(\frac{1}{4} - \frac{1}{9}\right) = 1.8886 \text{ eV}.$$

For ionization from $n = 1$,

$$\Delta E_{\infty 1} = 13.598 \text{ eV}.$$

$$\Delta E_{n \rightarrow m} = E_0 \left(\frac{1}{m^2} - \frac{1}{n^2}\right), \quad n > m$$

$$\lambda(\text{nm}) = \frac{1239.841984}{\Delta E(\text{eV})}.$$

2.5 orientations at level 2

one symmetric type + multiple directional orientations

2.5.1 symmetric type (as defined)

2.5.2 multiple directional orientations

Orientation 1 — aligned with beam (z-axis)

- axis of the structure is along the beam
- symmetric around beam
- no azimuthal variation

Orientation 2 — perpendicular (x-axis)

structure lies sideways relative to beam

- varies with ϕ

Orientation 3 — perpendicular (y-axis)

- same type as above
 - rotated 90° around beam
-

§3 Helium model – Alpha Structure

Start with anchor Hydrogen $R_{\text{cell}}=29447705.851$

Number of electrons in first shell $S_1=1$, State=1

$$E_{1,2} = R_{\text{cell},H} / (2 * \text{State}) + S_1 \times 1836 \times (816 + 64) = 24.5076$$

Number of electrons in first shell $S_1=0$, State=2

$$E_{2,3} = R_{\text{cell},H} / (2 * \text{State}) + S_1 \times 1836 \times (816 + 64) = 54.3937$$

3.1 Higher Energies

$$\boxed{\text{He}^+: E = -54.393736 \text{ eV}, r = 7,361,926.5 L_0}$$

Nucleus Curvature

$$k_{\text{CPP}} (2n + 2p) = (2 * 10)^2 + (2 * 6)^2 = 10,144$$

$$R_{\text{eff}} (k) = (51 k_{\text{CPP}}) L_0 = 51(10,144) L_0 = 517,344 L_0$$

The composite core for baryonic then becomes:

$$R_{\text{eff, baryonic}} = \frac{8}{51} 517,344 L_0 = 81,152 L_0$$

3.1.1 Energy Constant

For He^+ , the simplest first nuclear source is just:

$$S_\alpha = 2S_p = 2(288) = 576, \quad E_{\text{He}^+}(r) = \frac{576 \cdot 128}{r}$$

This is exactly **2× the hydrogen source strength** at the nucleus side. If He^+ has source $2S_H$, then:

$$E_{\text{He}^+}(r) = \frac{2S_H S_e}{r}, \quad \frac{E_{\text{He}^+}}{E_H} = 2 \frac{r_H}{r_{\text{He}^+}}$$

If atomic closure radius is inversely proportional to nuclear source strength, then doubling the source gives:

$$r_{\text{He}^+} = \frac{r_H}{2} \approx 941,980,608 \text{ cells}, \quad E_{\text{He}^+} = 4E_H = 4(13.598) = 54.392 \text{ eV}, \quad E_H = 13.598 \text{ eV}$$

$$E_{\text{He}^+} \approx 54.392 \text{ eV}$$

NIST lists the He II ground-state ionization energy as **54.417760 eV** for singly ionized helium.

Calculating the percent error

$$\frac{54.392 - 54.417760}{54.417760} \times 100 \approx -0.0473\%.$$

3.1.2 Energy Ladder

For He⁺, your simplest extension predicts:

$$E_n(\text{He}^+) = -\frac{4 \times 13.598}{n^2} = -\frac{54.392}{n^2} \text{ eV}$$

because the nucleus source doubled and the closure radius halved, giving a 4 × ground-state scale.

NIST gives the He II ground-state ionization energy as **54.417760 eV**, so your ground-state prediction **54.392 eV** is low by only about **0.047%**.

That means the whole He⁺ ladder becomes:

$$E_1 = -54.392 \text{ eV}, \quad E_2 = -13.598 \text{ eV}, \quad E_3 = -6.0436 \text{ eV}, \quad E_4 = -3.3995 \text{ eV}$$

3.1.3 He⁺ 2 → 1

Predicted:

$$\Delta E_{21} = 54.392 \left(1 - \frac{1}{4}\right) = 40.794 \text{ eV}, \quad \lambda \approx \frac{1239.841984}{40.794} \approx 30.39 \text{ nm} = 303.9 \text{ \AA}$$

NIST strong He II lines include 303.7804 Å and 303.7858 Å, which is an extremely tight match.

3.1.4 He⁺ 3 → 2

Predicted:

$$\Delta E_{32} = 54.392 \left(\frac{1}{4} - \frac{1}{9}\right) = 7.554 \text{ eV}, \quad \lambda \approx \frac{1239.841984}{7.554} \approx 164.1 \text{ nm} = 1641 \text{ \AA}$$

NIST lists strong He II lines around 1640.3321 Å, 1640.3447 Å, and nearby components, again a very close hit.

3.1.5 He⁺ 4 → 2

Predicted:

$$\Delta E_{42} = 54.392 \left(\frac{1}{4} - \frac{1}{16}\right) = 10.1985 \text{ eV}, \quad \lambda \approx \frac{1239.841984}{10.1985} \approx 121.6 \text{ nm} = 1215.9 \text{ \AA}$$

NIST strong He II lines include 1215.09 Å and 1215.17 Å.

Appendix A Helmholtz & Green

A.1 Helmholtz Equation?

The Helmholtz equation is:

$$\nabla^2\Psi + k^2\Psi = 0$$

It describes **steady-state wave behavior in space**.

It is what you get when a system oscillates sinusoidally in time, and you look only at the spatial part

If the full wave equation is:

$$\nabla^2\Psi - \frac{1}{c^2} \frac{\partial^2\Psi}{\partial t^2} = 0$$

and you assume harmonic time dependence:

$$\Psi(r, t) = \psi(r)e^{-i\omega t}$$

you obtain:

$$\nabla^2\psi + k^2\psi = 0$$

where:

$$k = \frac{\omega}{c}$$

That spatial equation is Helmholtz.

A.2 Green Function

A **Green function** answers one question:

If I place a point source at position \mathbf{r}_0 , what field does it create everywhere?

Mathematically:

$$(\nabla^2 + k^2)G(\mathbf{r}, \mathbf{r}_0) = -\delta(\mathbf{r} - \mathbf{r}_0)$$

The solution in 3D free space is:

$$G(r) = \frac{e^{ikr}}{4\pi r}$$

This is the **fundamental spherical wave** with a $1/r$ decay and Oscillatory phase term e^{ikr}

That is exactly the structure you already use:

$$\Psi(r) \propto \frac{\cos(kr - \omega t + \Phi)}{r}$$

So without naming it, you have already written the Helmholtz Green solution.

Appendix B – Curvatures and Phases

B.1 Neutron — Inner/Outer Shell Region (UUDZZZ core + EZZZ shell)

In this framework, **Neutrons** (UUDZZZ + EZZZ) are not independent primordial species; they arise dynamically when an outer Electron (EZZZ) curvature and phase layer with a Proton core UUDZZZ. The first non CPP Type region at the proton scale and that is the neutron. It is unstable in isolation. Its composition is a (UUDZZZ) core with a (EZZZ) outer shell.

It is unstable in isolation and decays in 882s but stable in the nucleus.

B.2 Neutron Curvature

$$R_{\text{eff}}(k) = (51 k_{\text{CPP}}) L_0 = 51(100) L_0 = 5100 L_0$$

The composite core for baryonic then becomes:

$$R_{\text{eff, baryonic}} = \frac{8}{51} 5100 L_0 = 800 L_0$$

B.3 Neutron phase descriptor $\mathcal{P}_N(\theta)$

$$\boxed{\mathcal{P}_n(\theta) = e \sin \theta}$$

B.4 Neutron Wave Function

Steady-state corridor disturbance fields emitted by each Region within the nucleus.

Appendix C – Proton Energy Ladder Closure

C.1 Locked Inputs (See Section 2)

C.2 Closure Residual Definition

Define the proton-side closure mismatch:

$$\Delta\phi_n = k_p r_n + \delta - (2n + 1)\pi$$

This quantity is then reduced modulo 2π into the interval:

$$-\pi < \Delta\phi_n \leq \pi$$

This produces the closure class.

C.3 Algebraic Reduction (key reproducible simplification)

From the locked ground-state solution:

$$a_0 = \left(16038 + \frac{1}{6}\right) \lambda_p$$

Therefore:

$$k_p a_0 = 2\pi \left(16038 + \frac{1}{6}\right)$$

At spectral radius $r_n = a_0 n^2$:

$$k_p r_n = 2\pi \left(16038 + \frac{1}{6}\right) n^2$$

All integer multiples of 2π drop out mod 2π , leaving:

$$\Delta\phi_n \equiv 2\pi \frac{n^2}{6} \pmod{2\pi}$$

This is the fully reduced reproducible formula.

C.4 Computation Rule (minimal form)

To compute the closure class for any n :

1. Compute $n^2 \pmod{6}$
2. Multiply by $2\pi/6$
3. Reduce into $(-\pi, \pi]$

C.5 Resulting Class Mapping

Because $n^2 \bmod 6$ can only be: {0, 1, 3, 4} the closure classes are:

$n^2 \bmod 6$	$\Delta\phi_n$
0	-60°
1	0°
3	$+120^\circ$
4	-180°

Equivalent direct rule: $\Delta\phi_n \approx 0 \Leftrightarrow n \equiv 1 \text{ or } 5 \pmod{6}$

C.6 Interpretation Boundary (do not overstate)

This diagnostic:

- Does **not** modify the hydrogen energy ladder.
- Does **not** introduce free parameters.
- Does **not** fit spectroscopy.

It is a deterministic compatibility class derived from:

- proton halo wavelength
 - triad phase offset
 - Bohr-scale separation
-

Appendix D – Electron Energy Ladder Closure

D.1 Locked Inputs (See Section 2)

- Electron-side phase offset **parameter**:
 - δ_e (to be chosen from locked eigen-phase assignments)
 - $\Phi_Z = 0$ and $\Phi_E = \pi/2$.

(So a reproducible choice is $\delta_e = \Phi_Z$ for triad-referenced closure or $\delta_e = \Phi_E$ for core-referenced closure.)

D.2 Closure Residual Definition (electron side)

Define the electron-side closure mismatch at radius r_n :

$$\Delta\phi_n^{(e)} = k_e r_n + \delta_e - (2n + 1)\pi$$

Reduce modulo 2π into:

$$-\pi < \Delta\phi_n^{(e)} \leq \pi$$

D.3 Exact reduction using the locked a_0 and λ_e

First express a_0 in units of λ_e :

$$\frac{a_0}{\lambda_e} = \frac{1,883,961,216}{52,224} = 36,074 + \frac{32,640}{52,224}$$

The remainder fraction reduces exactly:

$$\frac{32,640}{52,224} = \frac{85}{136} = \frac{5}{8}$$

So the locked identity is:

$$a_0 = \left(36,074 + \frac{5}{8}\right) \lambda_e$$

Therefore:

$$k_e a_0 = \frac{2\pi}{\lambda_e} \left(36,074 + \frac{5}{8}\right) \lambda_e = 2\pi \left(36,074 + \frac{5}{8}\right)$$

Modulo 2π , the integer term drops out:

$$k_e a_0 \equiv 2\pi \left(\frac{5}{8}\right) \pmod{2\pi}$$

D.4 Final reduced rule (thread-portable)

At $r_n = a_0 n^2$:

$$k_e r_n = k_e a_0 n^2 \equiv 2\pi \left(\frac{5}{8} n^2\right) \pmod{2\pi}$$

Also note:

$$(2n + 1)\pi \equiv \pi \pmod{2\pi}$$

So the electron-side closure residual reduces to:

$$\Delta\phi_n^{(e)} \equiv 2\pi \left(\frac{5}{8} n^2 \right) + \delta_e - \pi \pmod{2\pi}$$

This is the core reproducible result.

D.5 Discrete class structure (depends only on $n^2 \pmod{8}$)

Because only $n^2 \pmod{8}$ matters, and:

$$n^2 \pmod{8} \in \{0,1,4\}$$

the phase term $2\pi(5n^2/8)$ takes only three values:

- If $n^2 \pmod{8} = 0$: $2\pi(5n^2/8) \equiv 0$
- If $n^2 \pmod{8} = 1$: $2\pi(5/8) = \frac{5\pi}{4}$
- If $n^2 \pmod{8} = 4$: $2\pi(20/8) = \pi$

So the **electron closure classes** are:

$$\Delta\phi_n^{(e)} \in \left\{ \delta_e - \pi, \delta_e + \frac{\pi}{4}, \delta_e \right\} \pmod{2\pi}$$

Equivalently by n parity class:

- $n \equiv 0 \pmod{4} \Rightarrow n^2 \pmod{8} = 0 \Rightarrow \Delta\phi_n^{(e)} \equiv \delta_e - \pi$
- $n \equiv 2 \pmod{4} \Rightarrow n^2 \pmod{8} = 4 \Rightarrow \Delta\phi_n^{(e)} \equiv \delta_e$
- $n \text{ odd} \Rightarrow n^2 \pmod{8} = 1 \Rightarrow \Delta\phi_n^{(e)} \equiv \delta_e + \frac{\pi}{4}$

D.6 Two reproducible “locked-choice” examples for δ_e

The document gives the electron eigen-phase anchors: $\Phi_Z = 0$ and $\Phi_E = \pi/2$.

Case A — triad-referenced electron closure ($\delta_e = \Phi_Z = 0$)

$$\Delta\phi_n^{(e)} \in \{-180^\circ + 45^\circ, 0^\circ\}$$

- $n \equiv 2 \pmod{4}$: 0° (electron-side closure aligned)
- $n \equiv 0 \pmod{4}$: -180°
- $n \text{ odd}$: $+45^\circ$

Case B — core-referenced electron closure ($\delta_e = \Phi_E = \pi/2$)

$$\Delta\phi_n^{(e)} \in \{-90^\circ, +135^\circ, +90^\circ\}$$

D.7 Reproducibility summary (single-line formula)

Any future thread can reproduce the electron-side diagnostic by evaluating:

$$\Delta\phi_n^{(e)} = \left(\frac{2\pi}{52,224}\right)(1,883,961,216 n^2) + \delta_e - (2n + 1)\pi$$

then reducing mod 2π to $(-\pi, \pi)$.

The reduction will always collapse to the compact invariant:

$$\Delta\phi_n^{(e)} \equiv 2\pi\left(\frac{5}{8}n^2\right) + \delta_e - \pi$$

Appendix E – Combined ladder mapping diagnostics

E.1 Protium Energy Ladder

The protium system is not a new CPP closure. It is a steady-state two-Region corridor configuration whose equilibrium separation is determined by phase compatibility. In §2.2 the separation condition was obtained from

The Proton halo phase closure:

$$\Delta\phi_n^{(p)} = \left(\frac{2\pi}{\lambda_p}\right)(a_0 n^2) + \frac{2\pi}{3} - (2n + 1)\pi \equiv 2\pi\frac{n^2}{6}$$

The Electron halo phase closure:

$$\Delta\phi_n^{(e)} \equiv 2\pi\left(\frac{5}{8}n^2\right) + \delta_e - \pi$$

With this choice, the electron-side classes become:

- $n \equiv 0(\text{mod}4) \Rightarrow \Delta\phi_n^{(e)} = -90^\circ$
- $n \equiv 2(\text{mod}4) \Rightarrow \Delta\phi_n^{(e)} = +90^\circ$
- $n \text{ odd} \Rightarrow \Delta\phi_n^{(e)} = +135^\circ$

E.2 Combined closure-class table with $\delta_e = +90^\circ$ ($n = 1\dots 12$)

n	$n \text{ mod } 6$	Proton class	$\Delta\phi_n^{(p)}$	$n \text{ mod } 4$	Electron class	$\Delta\phi_n^{(e)}$	Pair (p, e)
1	1		0°	1		$+135^\circ$	$(0^\circ, +135^\circ)$

n	$n \bmod 6$	Proton class $\Delta\phi_n^{(p)}$	$n \bmod 4$	Electron class $\Delta\phi_n^{(e)}$	Pair (p, e)
2	2	-180°	2	+90°	(-180°, +90°)
3	3	+120°	3	+135°	(+120°, +135°)
4	4	-180°	0	-90°	(-180°, -90°)
5	5	0°	1	+135°	(0°, +135°)
6	0	-60°	2	+90°	(-60°, +90°)
7	1	0°	3	+135°	(0°, +135°)
8	2	-180°	0	-90°	(-180°, -90°)
9	3	+120°	1	+135°	(+120°, +135°)
10	4	-180°	2	+90°	(-180°, +90°)
11	5	0°	3	+135°	(0°, +135°)
12	0	-60°	0	-90°	(-60°, -90°)

The combined proton–electron closure structure repeats every 24 levels, the least common multiple of the mod-6 proton structure and mod-4 electron structure.

E.3 Energy ladder comparison (n = 1...8)

n	E_n^{std} (eV)	E_n^{calc} (eV)	ΔE(calc – std)	Closure class $\Delta\phi$ (deg)
1	-13.598000	-13.598000	0.000000	0°
2	-3.399500	-3.399500	0.000000	-180°
3	-1.510889	-1.510889	0.000000	+120°
4	-0.849875	-0.849875	0.000000	-180°
5	-0.543920	-0.543920	0.000000	0°
6	-0.377722	-0.377722	0.000000	-60°

n	E_n^{std} (eV)	E_n^{calc} (eV)	$\Delta E(\text{calc} - \text{std})$	Closure class $\Delta\phi$ (deg)
7	-0.277510	-0.277510	0.000000	0°
8	-0.212469	-0.212469	0.000000	-180°

E.4 Transition table with closure-class jumps ($\delta_e = +90^\circ$)

- Uses the locked **spectral ladder** $E_n = -13.598/n^2$ and $r_n = a_0 n^2$.
- Tags each level with your deterministic **proton closure class** $\Delta\phi_n^{(p)} \pmod{6}$.
- Tags each level with your deterministic **electron closure class** $\Delta\phi_n^{(e)}$ using $\delta_e = +90^\circ$ (core-referenced).

For frequencies I convert ΔE (eV) $\rightarrow f$ via the standard $f = \Delta E/h$.

$\Delta(\Delta\phi)$ is the signed smallest phase jump in degrees, reduced into $(-180^\circ, +180^\circ]$.

Transition	ΔE (eV)	f (THz)	$\Delta\phi_i^{(p)} \rightarrow \Delta\phi_f^{(p)}$	$\Delta(\Delta\phi^p)$	$\Delta\phi_i^{(e)} \rightarrow \Delta\phi_f^{(e)}$	$\Delta(\Delta\phi^e)$
2→1	10.198500	2465.986	-180° → 0°	-180°	+90° → +135°	-45°
3→2	1.888611	456.664	+120° → -180°	-60°	+135° → +90°	+45°
3→1	12.087111	2922.650	+120° → 0°	+120°	+135° → +135°	0°
4→2	2.549625	616.497	-180° → -180°	0°	-90° → +90°	-180°
4→1	12.748125	3082.483	-180° → 0°	-180°	-90° → +135°	+135°
5→2	2.855580	690.476	0° → -180°	-180°	+135° → +90°	+45°
5→1	13.054080	3156.463	0° → 0°	0°	+135° → +135°	0°
6→3	1.133167	273.998	-60° → +120°	-180°	+90° → +135°	-45°
6→2	3.021778	730.663	-60° → -180°	+120°	+90° → +90°	0°