Equation of the Universe

Atomic Nucleus

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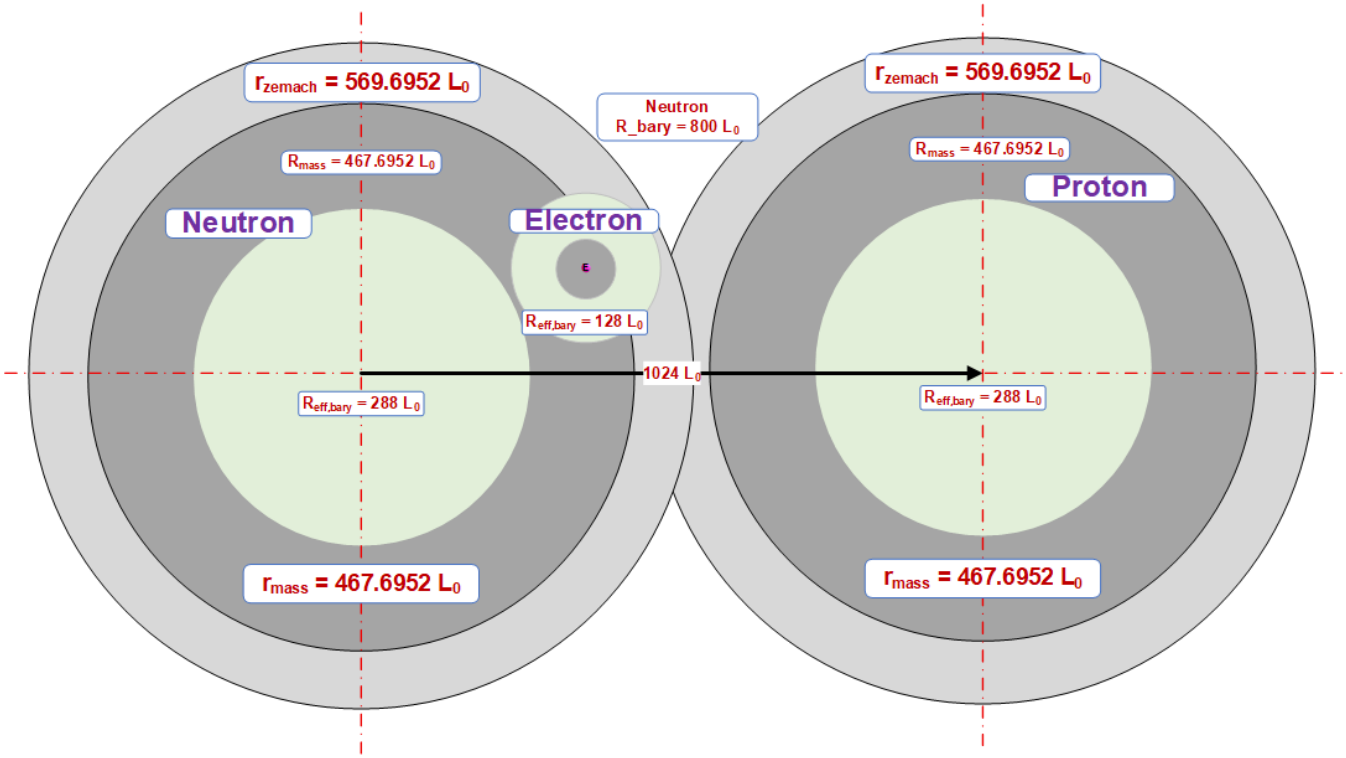
Equation of the Universe — Core Theory (Rev 3.44.0 or later)

§0 – Introduction

0.1 Definition

The nucleus is the stable multi-nucleon equilibrium formed when protons and neutrons enter a shared admissible corridor geometry. Within the EOTU framework, a nucleus is not an undifferentiated aggregate of nucleons and it is not a structure held together by an abstract force detached from geometry. It is a resolved interaction network in which productive proton–neutron corridors establish the binding of the whole assembly. Protons provide the stable curvature anchors of that network, while neutrons provide the additional shell participation required for extended nuclear closure. Nuclear binding therefore does not arise as an added property after formation. It is the energetic signature of the resolved corridor geometry itself.

That corridor geometry is expressed through a discrete spacing family. The standard nuclear binding grammar is governed by the three corridor classes d4, d5, and d6, which represent the stable engaged separation set for productive proton–neutron interaction across the solved packing network. These are not fitted labels applied afterward. They are the fixed geometric corridor family through which higher nuclear closure is counted, compared, and validated.



0.2 Conditions Required

A single proton defines the atomic nucleus limit, but extended nuclear structure begins with the proton–neutron pair. From that point forward, admissible nuclei are governed by productive proton–neutron nearest-neighbor corridors, geometric separations that preserve the mass-exclusion floor, and a compact packing geometry capable of stable closure. Proton–proton and neutron–neutron contacts may appear geometrically within the assembly, but they do not constitute productive binding corridors. The nucleus is therefore governed first by admissible geometry and only then by the energy associated with that geometry.

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§1 – Foundational Definitions

- Baryonic participation (Approximation from main Theory): $\frac{8}{51}$
- Baryonic participation (Planck 2018 base-ΛCDM value): 0.157303
- Energy map: $= \Gamma = 1$ eV SI bridge constant as the energy mapping scale (Derived in main Theory).
- Proton Zemach envelope radius: $r_Z = 569.6952\, L_0$(Derived in Proton Document)
- Engaged shell thickness: $Z_t = (51 - 8)\, L_0 = 43\, L_0$ (Derived in this Document)
- Nuclear separation: $d_{NN} \equiv d_{pn} \equiv 2r_{\text{nuc}}$ (Derived in this Document)

0.1A solve procedure

To solve any nuclei, follow the sequence below until a stable, admissible solution is obtained:

standard shorthand → graph solve → energy screen → minimal reinterpretation

- Standard shorthand is the default construction method using Tier-1 increment rules and the active backbone or reset anchor.
- If shorthand does not produce a clear or admissible solution, the structure should be evaluated using the contact graph derived from the relevant master closure.
- Admissible solutions must satisfy geometric closure. Energy agreement is used to validate among geometrically consistent configurations.
- When multiple admissible corridor partitions exist, preference is given to the configuration that:
  - remains closest to shorthand behavior
  - does not introduce new spacing families
- Minimal reinterpretation is applied only when required to resolve ambiguity and must preserve Tier-1 structure and spacing definitions.

If the standard growth rule, shorthand partition, or symmetry template does not produce a geometrically admissible configuration, then the nucleus shall be resolved directly from the master contact graph of the highest completed shell that contains it. The occupied nucleus is taken as an induced subgraph of that master graph, proton–neutron assignment is optimized on that subgraph, and corridor counts are derived only from actual nearest-neighbor contacts in the resolved geometry.

Fallback Rule for All Shell Layers

- Standard shell-growth rules and shorthand corridor increments are default construction guides, not absolute laws.
- When a standard construction fails to yield a compact, connected, geometrically admissible nucleus, the structure shall be re-derived from the master closed-shell contact graph for the relevant layer regime.
- The candidate nucleus is formed by selecting the occupied nodes as an induced subgraph of that master graph.
- Proton and neutron placement is then assigned to maximize valid productive p–n nearest-neighbor contacts subject to geometric admissibility and minimal distortion from the compact configuration.
- Corridor totals and spacing-family counts are taken only from the resolved graph, not from inventory alone.
- Energy is used only to validate among geometrically admissible solutions, not to override geometry.

0.1B Editorial Note — Working with CSV/Spreadsheet compilation

The model provides an independent predictive framework that can highlight inconsistencies or low-confidence measurements and may help guide refinement of experimental values.

SPECIAL: Case by case

BOUND: Nucleus remains bound for a specified time before decay

SEQUENCE_ROW: A non-physical but required structural member of an A-chain that preserves the deterministic progression of atomic number from Z=1 to Z=A. These rows are not targets for calculation and exist solely to maintain continuity of the construction sequence.

ANCHOR: This is where an element becomes an anchor for the following elements until a new anchor is established. At this point the energy is set to the anchor energy value and d4,d5,d6 reset values and continue growing begins from there.

RESONANCE: A resonance is:

- A measurable, short-lived intermediate state
- It exists briefly before decaying
- Shows up as a peak in scattering experiments

Observable consequence:

- Has a finite lifetime ($<10^{-21}$ s)
- Has a measured energy and width
- Still unbound, but detectable

UNBOUND: A nucleus is unbound if:

- It cannot exist as a stable system
- It immediately breaks apart into lighter nuclei
- Its energy is above the separation threshold

Observable consequence:

- Negative separation energy
- No bound state exists

FIRST OBSERVED: this is the first recorded measured value of the element of any type

NEUTRON RICH: this is where the element is confirmed to be neutron rich and d4 channel no longer increases only the d6 channel

1.2 corridor spacings rules

As nucleons are added, the nucleus does not grow through arbitrary accumulation. It grows through discrete packing geometry. The allowed corridor counts, the active spacing-family decomposition, and the closure behavior are determined by the occupied geometry of the nucleon network itself. In the standard solved form, the binding is read through the corridor family d4, d5, and d6 together with the productive corridor count supported by the admissible packing. Binding energy is then evaluated from that solved geometry. In this form, the nucleus is the first composite Region in which stable multi-body binding emerges directly from corridor participation, spacing-family structure, and geometric closure.

Nuclear binding is therefore determined entirely by:

- the number of engaged proton–neutron corridors
- the distribution of those corridors across (d_4, d_5, d_6)
- the geometric packing of nucleon Regions

The ordering is fixed:

Geometry → corridor counts (n_4, n_5, n_6) → binding

No continuous force law is introduced. The apparent interaction strength emerges from the discrete corridor set and their occupancy.

Growth of nuclear structure follows directly from corridor availability:

- Each proton–neutron pair introduces a coupled (d_4 + d_6) corridor
- Proton additions increase d_5 participation
- Neutron additions increase d_6 participation when no new d_4 corridors are available

Closed configurations maximize corridor engagement and produce enhanced stability. Open configurations remain admissible but with reduced corridor completeness.

Thus, the nucleus is a finite network of nucleon Regions whose edges are restricted to the invariant corridor set (d_4, d_5, d_6), and whose total binding is the scalar result of that discrete geometric structure.

The Alpha (α) configuration (A = 4) extends the first closed packing unit of nucleon Regions and establishes the geometric basis for larger nuclei. Subsequent nuclear growth proceeds through combinations and extensions of this unit:

- α structures act as stable geometric anchors within the packing network
- larger nuclei organize around α-derived cores and shells

    - corridor counts (n_4, n_5, n_6) evolve relative to these anchor configurations
- Milestone anchors occur at: $A = 4N_\alpha$
- Anchor Selection Rule
    - If loading is pure proton chain → Anchor = Protium
    - If loading has only one available p–n pair → Anchor = D (Deuteron)
    - If first alpha appears → Anchor = He-4_raw (24.4584 MeV), He-4 (28.2959 MeV)
    - If loading = 2α backbone → Anchor = Be-8 (56.5918 MeV)
    - If loading = 3α backbone → Anchor = C-12 (92.111181 MeV)
    - For more Alphas (see Spreadsheet EOTU_Nucleus_byElement_Z1_Z36_Vx.y)
    - If anchor does not match alpha count → Row is either: transitional (bridge) or incorrect
- Each anchor adds one α cluster to the preceding closure. The binding structure is governed by a fixed corridor set: (d_4, d_5, d_6)
- d4, d5, and d6 are determined first by geometry and corridor availability, and only then checked against measured energy.
- The scalar binding functional: $E_{\text{bind}} = N_\alpha E_\alpha + n_4 E_{d4} + n_5 E_{d5} + n_6 E_{d6}$

Standard Layer-2 and above corridor spacings

$$d_4 = 1024, \qquad d_5 = 1024 + 2\,(16) = 1056, \qquad d_6 = 1024 + 43 = 1067$$

$$E_{d4}(1024) = 2.0780644 \text{ MeV}, \qquad E_{d5}(1056) = 0.9892874 \text{ MeV}, \qquad E_{d6}(1067) = 0.6735958 \text{ MeV}$$

$$E_{\text{bind}} = N_\alpha E_\alpha + n_4 E_{d4} + n_5 E_{d5} + n_6 E_{d6},$$

$$E_\alpha = 28.2959 \text{ MeV}$$

Growth behavior

- Each p-n pair contributes d4+d6
- Proton additions increase only d5. d5 remains unchanged under neutron addition.
- Neutron additions without a proton corridor increase d6.
- For a given anchor chain growth is limited by the available nearest-neighbor outer p–n pairs.
- Each outer nucleon forms productive p–n corridors with available opposite-type nearest neighbors.
- The number of outer p–n corridors equal the number of nearest-neighbor proton–neutron pairs in the outer shell.
- d6 should be greater than d4 based on neutron cap growth as a pair.

contribution	**corridors**	**Energy Form**	**Layer 2 Δ growth N+1**	**Layer 3 Δ growth N+1**
Each p-n corridor	$d_4 + d6$	$E_{d4}(1024) + E_{d6}(1067)$ 2.0780644 +0.6735958	+1, 0, +1	+1, 0, +1
Each neutron extra	d_6	$E_{d6}(1067)$	0,0,1	0,0,1
Each proton cap (p < 3)	$2d_5$	$2E_{d5}(1056)$ 1.9785748 MeV	0, +2, 0	0, +2, 0
proton-rich (extra p ≥ 3)	$n_5\ d_5$	$1/2E_{d5}(1056)$		

contribution	corridors	Energy Form	Layer 2 Δ growth N+1	Layer 3 Δ growth N+1

The discrete corridor spacings (d4, d5, d6) emerge as the stable integer-aligned limits of the Layer-2 and Layer-3 compression solutions. Once these values are reached, further averaging collapses and only discrete corridor counts remain valid.

These three separations define the standard corridor energies used throughout all nuclei calculations. They are **derived from baryonic displacements within the Zemach shell**.

0.2.1 Calculating Percent Error

$$\%\, error = \frac{E_{calc} - E_{meas}}{E_{meas}} \times 100$$

02.2 Standard Layer-1 corridor spacings

The compression from 1021.3904 → 935.3904 increases the individual corridor energy from ~2 MeV to ~6 MeV, which in the closed helium-4 network produces the observed ~28 MeV total binding.

$$d_1(^2\mathrm{H}) = d_{NN} = 1021.3904\, L_0, \qquad E_{pn}(1021.3904) \approx 2.1770 \text{ MeV}$$

Minimum Overlap

$$\boxed{E_{pn}(1136) \approx 3.35 \times 10^{2} \text{ eV} = 3.35 \times 10^{-4} \text{ MeV}}$$

1.3 Nuclear Coupling Principle

1.3.1 Mass-exclusion floor

The nucleon mass boundary provides a strict non-overlap constraint

$$r_{mass} = 467.6952\, L_0$$

so that the minimum admissible center separation is

$$d_{min} = 2r_{mass} = 935.3904\, L_0.$$

No nucleon pair can stably exist at separations below this value.

1.3.2 Nuclear separation

The fundamental nuclear interaction in this framework is the **primary proton–neutron pair**. Productive p–n corridors in larger nuclei therefore represent realizations of this primary interaction, and nuclear binding arises from the networks of such p–n pairs whose interactions are mediated through the p–n corridor graph determined by nucleon packing geometry. Once formed, a proton–neutron pair behaves as a primary interaction unit within the nuclear network. Larger nuclei therefore consist of networks of such p–n pairs whose interactions are mediated through the corridor graph.

The interaction energy between a proton and neutron separated by distance d is given by $E_{pn}(d)$.

$$E_{\text{nucleus}} \approx \sum_i E_{pn}\,(d_i) + \sum_{i<j} E^{ij}_{\text{pair–pair}}$$

Therefore higher layers (Layer-2, Layer-3…) nuclei become naturally describable as

$$\mathrm{N_{pairs}} \approx \frac{\mathrm{A}}{2}$$

with the interactions between these pairs determined by the p–n corridor graph produced by nucleon packing geometry. The deuteron therefore represents the simplest realization of the primary p–n interaction.

Nuclear separation is governed by the interacting curvature shell inside the Zemach boundary where corridor-mediated curvature relaxation occurs.

Each nucleon contains:

- Proton Mass boundary: $r_{\text{mass}} = 467.6952\, L_0$
- Proton Zemach boundary: $r_{\text{Zemach}} = 569.6952\, L_0$
- Proton Zemach layer thickness is: $r_{\text{zemach}} - r_{\text{mass}} = 102\, L_0$

The Zemach layer consists of two 51-thick curvature shells (one per interacting triad); of each 51-layer, 8 L_0 is baryonic-active and the remainder curvature-closure.

$$Z_t = 51 - 8 = 43$$

Nuclear contact radius: Each nucleon contributes one 51-cell curvature propagation shell. Of the 51 cells, 8 correspond to baryonic participation, leaving 51 − 8 = 43 cells available for corridor curvature interaction.

Thus the nuclear interaction radius is determined not by the nucleon's geometric size alone but by the number of curvature degrees of freedom available outside the baryonic core.

$$r_{\text{nuc}} = r_{\text{mass}} + Z_t = 467.6952 + 43 = 510.6952\, L_0$$

Symmetric nucleon separation (baseline nuclear spacing)

$$d_{NN} \equiv 2r_{\text{nuc}} = 1021.3904\, L_0$$

Using the updated coherence length:

$$1\, L_0 = 0.001797965855 \text{ fm}$$

$$d_{pn} = 1021.3904 \times 0.001797965855 \text{ fm} = 1.8364250638 \text{ fm} \approx 1.8364 \text{ fm}$$

This lies within the empirical deuteron separation scale (~1.84 fm), indicating:

- Mass envelopes do not overlap.
- Active curvature shells nearly touch.
- Zemach envelopes overlap shallowly, forming the nuclear corridor.

The characteristic ~1.8 fm nucleon spacing observed throughout nuclear physics emerges naturally from the corridor interaction geometry which lies inside the Zemach envelope but outside the mass shell.

Two interacting nucleons therefore stabilize at a separation $d_{NN} = 2r_{nuc} \approx 1.8364$ fm. This distance represents the point at which curvature-shell engagement is maximized while mass-shell overlap remains forbidden. Because the same shell geometry governs all nucleons, multi-nucleon systems naturally cluster around the same ~1.8 fm nearest-neighbor spacing.

1. **Primary pair energy** — the deuteron-scale base p–n bond.
2. **Corridor count** — the number of productive p–n contacts in the solved packing graph.
3. **Geometric closure energy** — the additional stabilization produced by closed nuclear packing.

A nucleus therefore does not bind by corridor count alone. It binds through the combination of primary p–n pairing and the geometric reinforcement of the closed nucleon network. An n–n pair does not form an independently productive primary corridor, but in the presence of a proton and sufficiently compact geometry, the n–n side participates in cooperative curvature minimization of the full three-body nucleus.

1.3.3 Helium as Alpha

Helium-4 Geometry: In the four-nucleon system, the nucleons compact into the regular tetrahedral closure until limited by the mass-shell exclusion boundary and the nuclei are well within the Zemach shell so it is not part of the coupling however the **axial projection of the zeteon triad remains**, introducing a residual geometric offset even in the compact tetrahedral closure.

$$r_{mass} = 467.6952\,L_0, \qquad Z_t = 1\,L_0, \qquad \beta = 30^\circ$$

$$d_1(^4He) = 2r_{mass} + Z_t \cos\,\beta$$

$$d_1(^4\mathrm{He}) = 2r_{mass} + \cos(30^\circ) = 2(467.6952) + 0.8660 \;\; = 936.2564\,L_0$$

$$d_{d1}(^4\mathrm{He}) = 936.2564\,L_0$$

Total Binding Energy

$$E_{d1}(936.2564) \approx \; 6.1146 \text{ MeV}$$

$$E_{raw}(^4\mathrm{He}) = \sum E_{d1} = 4E_{pn}(936.2564) = 4(6.1146) \approx 24.4584 \text{ MeV}$$

Cooperative shell enhancement

$$E_{bind}(^4\mathrm{He}) = \Gamma\, S_{closure}\, E_{raw} = \Gamma\left(1+\frac{8}{51}\right)(24.4584) \approx 28.2959 \text{ MeV}$$

Measured Value

$$E_{bind}(^4\mathrm{He}) = 28.30 \text{ MeV}, \qquad \Delta E = 28.30 - 28.30 = 0.00 \text{ MeV}$$

$$\%\,error = \frac{0.0}{28.30} \times 100 \approx 0.0\%$$

1.4 Core Nucleon Count and Core Layer Index $L_{core}(Z)$

For a first-pass balanced nucleus in EOTU (valid across Z = 1–118), we approximate the total nucleon count as Proton count: $n_p = Z$ and Neutron count: $n_n \approx Z$ so that:

$$N_{\mathrm{tot}}(Z) \approx n_p + n_n \approx 2Z.$$

1.4.1 The layer capacity rule

The layer capacity rule arises from the geometry of successive cubic lattice shells surrounding the central nucleon. Consider nucleons arranged on a cubic lattice with the central nucleon occupying layer $L = 0$. The set of lattice sites at layer Lforms the outer surface of a cube of side length $2L + 1$. The number of sites added when expanding the cube from side $2L - 1$ to $2L + 1$ is therefore

$$(2L + 1)^3 - (2L - 1)^3$$

Expanding the cubes gives

$$(2L + 1)^3 - (2L - 1)^3 = 24\,L^2 + 2$$

However, only the sites lying on the principal packing directions contribute to stable nucleon centers in the EOTU packing construction. These correspond to the four symmetry directions defined by the corridor

interaction geometry. When restricted to these allowed positions, the effective number of new nucleon sites introduced at layer L becomes

$$\Delta C_L = 4L^2.$$

The cumulative capacity of the packing therefore becomes

$$C_{tot}(L) = \sum_{k=1}^{L} 4\,k^2 = \frac{2}{3}L(L+1)(2L+1)$$

which defines the number of nucleons that can be accommodated within L packing layers of the nucleus.

Layer capacity Table

Layer	Nucleons	C_tot(L)	End Element	Atomic Number
1	4	4	He	He-2
2	16	20	Neon	Ne-10
3	36	56	Nickel	Ni-28
4	64	120	Neodymium	Nd-60
5	100	220	Darmstadtium	Ds-110
6	144	364		

The core layer index is then defined by

$$L_{\text{core}}(Z) = \min\{L: N_{\text{tot}}(Z) \le C_{\text{tot}}(L)\}.$$

0.4.2-layer Compression

A nuclear shell penetrates inward below the universal nucleon spacing d_{NN} because the added outer-layer population occupies a finite fraction of the total engaged cluster and therefore engages a corresponding fraction of the available engaged Zemach-layer thickness Z_t.

For a completed shell this produces the symmetric closed-shell limit. For a partially occupied shell the same principle applies, with the occupied outer-layer population replacing the full shell population.

Using $\Delta C_L = 4L^2$ and $C_{tot}(L) = \frac{2}{3}L(L+1)(2L+1)$

General compression rule: The inward penetration of the shell is proportional to the fraction of the cluster represented by the occupied outer layer:

$$\delta_L = Z_t \frac{\Delta C_{occ}}{C_{tot}(L)}$$

Closed-shell limit: For a completed shell

$$\Delta C_{occ} = \Delta C_L$$

Therefore, the geometric fraction contributed by the outer shell is

$$f_L = \frac{\Delta C_L}{C_{tot}(L)}$$

and the inward penetration becomes

$$\delta_L = Z_t f_L = Z_t \frac{\Delta C_L}{C_{tot}(L)}$$

Closed-shell nearest-neighbor spacing: Substituting into the spacing relation gives

$$d_L = d_{NN} - Z_t \frac{\Delta C_L}{C_{tot}(L)}$$

Hence the closed-shell nearest-neighbor spacing becomes

$$d_L = d_{NN} - \delta_L = d_{NN} - Z_t \frac{\Delta C_L}{C_{\text{tot}}(L)} = d_{NN} - Zt \frac{6L}{(L+1)(2L+1)}.$$

Using d_{NN} = 1021.3904 L0 and Zt = 43 L0, the inward penetration for the first few shells is:

- L = 1: δ_L = 43.0000 L_0, so d1 = 981.0000 L_0
- L = 2: δ_L= 34.4000 L_0, so d2 = 986.9904 L_0
- L = 3: δ_L = 27.642857 L_0, so d3 = 993.747543 L_0
- L = 4: δ_L= 22.448276 L_0, so d4 = 1001.551724 L_0

0.4.3 Nucleon Neighbor Graph and Corridor Candidates

The geometric packing defined in Section 0.5 determines the set of nucleon contacts within a nucleus. These contacts form the **nucleon neighbor graph**. For a nucleus containing A ∈ ℕ nucleons, define

$$E(A)$$

as the **contact graph of the equal-radius nucleon packing**, where

- vertices represent nucleons, and
- edges represent nearest-neighbor contacts between nucleon regions.

Two nucleons i and $j$are considered neighbors if their centers satisfy

$$d_{ij} \leq 2r_{pn}.$$

Adding a neutron or proton

0.4.4 Proton–Neutron Corridor Candidates

Binding corridors arise only between proton–neutron pairs. Proton–proton and neutron–neutron contacts do not produce an engaged corridor. Let

$$\ell(i) \in \{p, n\}$$

denote the nucleon type (proteon or neutreon) of vertex i. The set of proton–neutron corridor candidates is defined as

$$\mathcal{L}_{pn}(A, Z) = \{(i, j) \in E(A) : \ell(i) = p,\ \ell(j) = n\}.$$

These edges represent the dormant-corridor interaction pathways available for nucleon binding.

$$N_{pn}(A, Z) = |\mathcal{L}_{pn}(A, Z)|$$

The number of engaged corridors in a nucleus is therefore $N_{pn} =| \mathcal{L}_{pn}(A, Z) |$, where the corridor set $\mathcal{L}_{pn}$ is obtained directly from the nucleon packing graph.

0.4.5 non-pair nuclear compression

- A primary p–n pair forms the initial nuclear corridor.
- Introduction of the second neutron does not create a second independent deuteron-like bond.
- Instead, the three-body configuration compacts inward to further minimize total curvature.
- In that compact state, the n–n side becomes geometrically relevant through proton mediation, even though it is not a primary standalone corridor.
- The observed tritium binding energy is therefore the signature of a cooperative three-body curvature minimum, not a simple sum of two-body links.

Tritium contains two proton–neutron corridors and no proton–proton constraint. The three-body geometry therefore permits inward corridor compaction until the engaged shell thickness and intrinsic corridor angle β establish the minimum curvature configuration consistent with the nucleon mass boundary.

$$d_{pn}(^3H) = 2r_{mass} + Z_t \cos\ \beta$$

where β is the corridor half-angle of the equilateral triangular nucleon configuration (β = 30°).

0.4.6 pair-exclusion expansion

- A primary p–n pair remains the fundamental productive nuclear corridor.
- In a three-body Layer-1 nucleus containing a proton–proton adjacency, the p–p side does **not** form a productive corridor.
- Because the p–p side cannot support the same cooperative compaction as the tritium *pnn*geometry, it acts as a geometric exclusion side.
- This exclusion increases the equilibrium p–n spacing relative to the non-pair compressed tritium configuration.

- The lower binding of helium-3 therefore arises not from fewer productive p–n corridors, but from increased corridor spacing imposed by the proton–proton side.

In a three-body Layer-1 nucleus containing a p–p adjacency, the proton–proton side does not form a productive corridor and imposes a minimal geometric exclusion. Relative to the non-pair compressed tritium geometry, this exclusion increases the equilibrium p–n spacing by one baryonic step:

$$d_{pn}(^3\mathrm{He}) = d_1(^3\mathrm{H}) + 8L_0$$

where $8L_0$is one baryonic lattice step of the nucleon packing metric.

The resulting nucleus contains the same two productive p–n corridors as tritium, but at larger separation and therefore lower total binding energy.

0.4.7 closed tetrahedral compaction

- In the balanced four-nucleon Layer-1 nucleus, the nucleons occupy the regular tetrahedral closure.
- All six nearest-neighbor contacts are geometrically equivalent.
- The fully closed three-dimensional corridor network removes the residual open-direction freedom present in the three-body nuclei.
- The compact configuration therefore proceeds until limited by the nucleon mass-shell non-overlap boundary.

The limiting condition occurs when neighboring nucleon mass shells just touch, establishing the equilibrium separation

$$d = 2r_{mass}.$$

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§2 – Binding Energy

The nuclear binding energy for elements 0 through 24 (Neutron to Chromium) generally increases with atomic number as the nucleus becomes more massive and more nucleons are bound together. The binding energy per nucleon (an indicator of stability) starts very low, spikes sharply at Helium-4, and steadily increases toward Iron-56.

Key Isotopes Binding Energy per Nucleon (0-24)

- 0 (Neutron): 0 MeV
- 1H (Hydrogen): 0 MeV (1 nucleon)
- 2H (Deuterium): 1.112 MeV
- 3He (Helium-3): 2.573 MeV
- 4He (Helium-4): 7.074 MeV (Significant stable isotope)
- 6Li (Lithium): 5.332 MeV
- 9Be (Beryllium): 6.463 MeV
- 11B (Boron): 6.928 MeV
- 12C (Carbon): 7.68 MeV
- 14N (Nitrogen): 7.476 MeV
- 16O (Oxygen): 7.976 MeV
- 19F (Fluorine): 7.779 MeV
- 20Ne (Neon): 8.032 MeV
- 23Na (Sodium): 8.111 MeV
- 24Mg (Magnesium): 8.261 MeV
- 27Al (Aluminum): 8.332 MeV
- 28Si (Silicon): 8.448 MeV
- 31P (Phosphorus): 8.481 MeV
- 32S (Sulfur): 8.493 MeV
- 35Cl (Chlorine): 8.52 MeV
- 40Ar (Argon): 8.595 MeV
- 40Ca (Calcium): 8.551 MeV
- 45Sc (Scandium): 8.619 MeV
- 48Ti (Titanium): 8.723 MeV
- 51V (Vanadium): 8.742 MeV
- 52Cr (Chromium): 8.776 MeV

$$E_{pn}(1084) \approx 0.35830 \text{ MeV}$$

2.1 Universal nuclear binding evaluation rule.

This expression represents the universal proton–neutron corridor binding functional. Binding energies of specific nuclei are obtained by evaluating the functional at the separations imposed by the derived nuclear geometry and summing over the allowed proton–neutron corridors. Nuclear binding arises exclusively from proton–neutron corridor engagement. Proton–proton and neutron–neutron adjacency does not form an engaged corridor and therefore contributes no binding energy.

For any nucleus, the binding calculation proceeds in two stages. First, compute the raw productive-corridor energy

$$E_{\text{raw}} = \sum_{(i,j) \in \mathcal{L}_{pn}} E_{pn}\left(d_{ij}\right)$$

Where:

- $\mathcal{L}_{pn}$ denotes the set of productive proton–neutron corridors.
- $E_{pn}(\text{d})$ denotes the proton–neutron corridor energy functional evaluated at separation d.

For derivation context only, the effective spacing formulation is given by:

$$E_{\text{raw}} = N_{pn} E_{pn}\left(d_{avg}\right).$$

2.2 Cap/exposure coherence (Derived in Appendix)

$$\alpha(d) = \arccos\left(\frac{d}{2r_Z}\right), \qquad F(d) = \frac{\alpha(d)}{\theta_0}, \theta_0 = \frac{\pi}{6}$$
$$g_{pn}(d) = 1 + 2\cos\left(\frac{\pi}{4}\right) F(d), \qquad \Delta g_{pn}(d) = g_{pn}(d) - 1$$

where $\Delta g_{pn}(d)$ is the coherence term derived in Appendix E.

2.3 Shell–shell corridor overlap volume

Define the equal-sphere lens overlap volume:

$$V_{\text{lens}}(r, d) = \frac{\pi\,(4r + d)\,(2r - d)^2}{12} (d \le 2r)$$

Then the **engaged corridor shell** overlap is the difference of outer and inner lenses:

$$V_{\text{shell}}(d) = V_{\text{lens}}(r_Z, d) - V_{\text{lens}}(r_Z - t, d)$$

2.4 Proton–Neutron Corridor Binding Functional

$$\boxed{E_{pn}(d) = \Gamma\left(\frac{8}{51}\right) V_{\text{shell}}(d)\, \Delta g_{pn}(d)}$$

(Units: if r_Z, d, t are in L_0, then V_{shell} is in L_0^3, and Γ is your existing "energy per engaged CPP corridor volume" mapping.)

The rise and fall of nuclear binding energy per nucleon follows directly from the corridor-coupling mechanism. As nucleons are added to a nucleus, the number of proton–neutron corridors increases, providing additional curvature-relaxation channels and increasing the total binding energy.

In mid-mass nuclei the packing geometry approaches maximal corridor connectivity, where each nucleon participates in the largest number of efficient corridors. This configuration occurs near the iron–nickel region (A ≈ 56–62), producing the highest binding energy per nucleon.

For larger nuclei, the required neutron excess and increasing nuclear radius reduce effective corridor engagement. The binding efficiency therefore declines. The empirical iron peak thus arises naturally from the geometry of the proton–neutron corridor network rather than from a separate nuclear force mechanism.

$$E_{bind} \propto N_{pn}\, V_{shell}(d)\, \Delta g_{pn}(d)$$

Binding therefore scales with the number of engaged proton–neutron corridors and the geometric efficiency of their shell overlap, causing the binding energy per nucleon to increase until corridor connectivity saturates in the iron–nickel region.

Because the nuclear spacing is stabilized near $d_{NN} = 2r_{nuc}$, the corridor functional contributes nearly a fixed value per engaged nearest-neighbor proton–neutron corridor. The binding energy per nucleon therefore scales primarily with the number of engaged p–n corridors per nucleon, which is a geometric coordination problem in a fixed-spacing packing. In 3-D close packing the maximum neighbor coordination is 12, but finite clusters fall below this value due to surface loss.

2.5 – Geometric Closure

2.5.1 Geometric closure requirement

- Structural determination must be resolved through the **contact topology** of the α-cluster network.
- Valid nuclei correspond to **induced subgraphs** of the fixed tetrahedral coordinate lattice.
- Admissible configurations must satisfy:
  - full network connectivity
  - compactness (no artificial extensions)
  - exact α-cluster decomposition

2.5.2 Discrete cap-completion behavior

- Growth beyond a given closure proceeds through **discrete geometric additions**, not continuous rearrangement.
- Available configurations are restricted to a finite set of **symmetry-equivalent placements** within the fixed lattice.
- Structural evolution follows a **quantized completion sequence**, with each step corresponding to the addition of a geometrically admissible α-cluster unit.

2.5.3 Geometric constraint on corridor partition

- While the **total corridor count** is preserved across equivalent configurations, the finer decomposition:

  $$(n_4,\ n_5,\ n_6)$$

  is constrained by the discrete interface structure of the α-cluster network.

- Contributions arising from multi-contact interfaces occur in **quantized increments**, imposing parity constraints on admissible partitions.
- As a consequence:
  - energy-optimal corridor combinations may be **geometrically inadmissible**
  - the realized configuration is the **nearest geometrically valid partition**, not necessarily the scalar energy minimum

---

2.5.4 Structural implication

- At higher A, the governing hierarchy becomes:

  $$\text{Geometry} \rightarrow \text{allowed corridor partitions} \rightarrow \text{binding energy}$$

- The α-cluster network therefore acts as a **discrete constraint system**, governing the fine structure of nuclear binding beyond the energy screen.

§3 – Layer 2 Nuclei A ≥ 5 and A ≤ 20

3.1 Layer 2 structural rules govern nuclei

The following structural rules govern nuclei in the Layer-2 regime.

- In general, additional nucleons occupy outer Layer-2 positions around an ^{4}He core but is not required for all nuclei. There are special cases
- Productive nuclear binding arises from p–n corridors.
- p–p and n–n contacts are geometric and nonproductive, though they influence shell stress and geometry.
- Beginning at A = 8, nuclei may follow two structural branches:
  - Single-core shell continuation
    Additional nucleons occupy Layer-2 positions around the ^{4}He core.
  - Two-alpha clustered backbone
    Two closed ^{4}He substructures form a paired alpha system.
  - In the alpha-backbone branch, neutron bridges may stabilize the alpha pair.

3.2 Layer 2 packing

- It contains up to an additional 16 nucleons, giving the first closed structure at $A = 4 + 16 = 20$.
- The shell forms through face-anchored tetrahedral growth on the four triangular faces of the Layer-1 tetrahedron.
- Each face receives a cap nucleon, which stabilizes three additional nucleons along the edges of the face.
- Across the four faces this produces $4 \times 4 = 16$ Layer-2 nucleons arranged symmetrically around the core.
- The full geometric derivation is given in Appendix B.

3.2.1 Evaluation of Partially Filled Layer-2 Nuclei

For any Layer-2 element, shell compression may be evaluated directly from the general **partial-shell law** using the occupied outer-layer population. Exact productive-corridor counts and full binding-energy evaluation require the contact graph corresponding to the occupied Layer-2 sites.

When the configuration is not a locked symmetry case, the graph is obtained as the **induced subgraph** of the Layer-2 closure graph derived in §3.1. Layer-2 nucleons pack around the tetrahedral Layer-1 core through face-anchored tetrahedral growth.

The Layer-2 closure graph derived above defines the **master geometry**. Partially filled Layer-2 nuclei are evaluated as induced subgraphs of this structure.

Geometric Contact Counts: The complete **A = 20** packing contains three classes of contacts.

Layer-1 internal contacts

- The tetrahedral Layer-1 core contains $E_{11} = 6$ contacts.

Layer-1 → Layer-2 contacts
For each triangular face patch:

- The face-cap nucleon contacts the **3 Layer-1 nucleons** of that face
- Each of the **3 edge nucleons** contacts the **2 Layer-1 nucleons** forming its edge
- Per face: $3 + (3 \times 2) = 9$

Across four faces:

$$E_{12}^{\text{face}} = 3 + (3 \times 2) = 9, \qquad E_{12} = 4 \times 9 = 36$$

Layer-2 internal contacts
Within each face patch:

- Cap–edge contacts = 3
- Edge–edge contacts = 3
- Per face: 6
- Across four faces: 24
- Additional contacts occur between edge nucleons belonging to adjacent faces that share a Layer-1 edge.

There are **6 Layer-1 edges**, producing **6 additional Layer-2 contacts**

$$E_{22} = 24 + 6 = 30$$

Total Geometric Contacts

$$E_{total} = E_{11} + E_{12} + E_{22} = 6 + 36 + 30 = 72$$

Proton–Neutron Assignment

For the closed Layer-2 shell the balanced composition is

$$A = 20, \qquad Z = 10, \qquad N = 10$$

The proton–neutron labeling is chosen to maximize p–n nearest-neighbor pairs subject to the geometric packing rules.

The optimized configuration produces 48 p–n contacts

These represent the **productive binding corridors** for the $A = 20$ closed shell.

Result

The first full outer shell therefore consists of

- Layer-1 tetrahedral core (4 nucleons)

- Layer-2 face-growth shell (16 nucleons)
- forming a **20-nucleon closed packing structure** with **48 productive p–n binding corridors**.

The $A = 20$ closure derived above defines the complete geometric contact graph of the Layer-2 structure surrounding the tetrahedral Layer-1 core. This closure graph serves as the **master Layer-2 contact graph** for all nuclei satisfying

$$5 \leq A \leq 20.$$

Partially filled Layer-2 nuclei are **not derived from a new geometric construction**. Instead they are evaluated as **induced subgraphs** of the solved Layer-2 closure graph.

For a nucleus with $A < 20$, the evaluation procedure is:

1. Select the occupied Layer-2 sites
2. Retain contacts whose endpoints are occupied
3. Compute the resulting contact counts
4. Assign Zprotons and Nneutrons to maximize p–n neighbor pairs

yielding the total productive corridor count

$$N_{pn} = N_{pn}^{(11)} + N_{pn}^{(12)} + N_{pn}^{(22)}.$$

This corridor count is then used in the binding-energy calculation.

3.3 Layer-2 Metric Structure

For $L = 2$the lattice rule $i + j + k \leq 3$ yields the **20-node tetrahedral cluster** corresponding to the closed Layer-2 nucleus.

The geometric derivation above determines the **contact graph** of the $A = 20$ nucleus but does not yet specify the **metric spacing** between all nucleon pairs.

Three classes of geometric contacts exist

$$E_{11} = 6, E_{12} = 36, E_{22} = 30$$

where

• E_{11}= Layer-1 core contacts
• E_{12}= Layer-1 → Layer-2 contacts
• E_{22}= Layer-2 shell contacts

For the balanced configuration

$$Z = 10, \qquad N = 10$$

the optimized proton–neutron labeling produces the productive-corridor decomposition

$$N_{pn}^{(11)} = 4, \qquad N_{pn}^{(12)} = 24, \qquad N_{pn}^{(22)} = 20$$

so that

$$N_{pn,tot} = 48.$$

The Layer-1 → Layer-2 contacts correspond to the compressed mixed-shell separation

$$d_{12} = d_{NN} - 34.4 = 986.9904\, L_0.$$

Thus the Layer-2 nearest-neighbor spacing set becomes

$$d_{11} = d_{NN}, d_{12} = 986.9904\, L_0, d_{22} = d_{NN}.$$

---

§4 – Layer 3 Nuclei A ≥ 21 and A ≤ 56

Not used for direct binding evaluation

Layer-3 represents the third geometric packing shell of the EOTU nuclear region. It is constructed by expanding the solved Layer-2 tetrahedral surface with an additional triangular packing layer on each face.

This shell adds

$$\Delta C_3 = 4L^2$$

with $L = 3$, giving

$$\Delta C_3 = 36$$

additional nucleons.

The full Layer-3 closed shell therefore contains

$$A = 4 + 16 + 36 = 56$$

nucleons. This structure corresponds to the complete three-layer tetrahedral packing of equal spheres.

4.1 Layer 3 packing

- Layer 3 forms the second complete shell surrounding the tetrahedral Layer-1 and Layer-2 core.
- It contains 36 nucleons, giving the next closed structure at A = 56.
- The shell capacity follows the geometric packing rule $\Delta C_l = 4L^2$.
- For $L = 3$ the Layer-3 shell contains 36 nucleons.
- Layer-3 nucleons pack through continued tetrahedral face-growth on the Layer-2 shell.
- Each triangular surface region of the Layer-2 shell expands outward, forming a nine-node triangular surface patch (6 lower + 3 upper)
- These patches extend the tetrahedral symmetry established by the Layer-1 and Layer-2 shells.
- The resulting shell therefore contains 36 Layer-3 nucleons arranged symmetrically around the 20-nucleon inner core.
- The full closed structure contains $A = 56$ nucleons.
- Proton–neutron labeling continues to favor p–n nearest-neighbor contacts, with extra neutrons appearing preferentially in the outer shell as curvature-stabilizing sites.

The Layer-3 shell is constructed by adding a third triangular packing layer to each of the four tetrahedral faces of the solved Layer-2 nucleus.

The Layer-2 nucleus contains:

- Layer-1 core: 4 nucleons (tetrahedron)
- Layer-2 shell: 16 nucleons arranged across four triangular faces

Layer-3 extends this structure by adding an additional triangular patch to each face. Each face contributes

$$L^2 = 3^2 = 9$$

new nucleons.

4.2 Layer-3 Metric Structure

The Layer-3 geometric structure is derived in Appendix C from the tetrahedral close-packing lattice using the $A = 56$ closed shell. The resulting contact graph serves as the master geometric structure for all Layer-3 nuclei.

The closed-shell node populations are

$$C_1 = 4, C_2 = 16, C_3 = 36.$$

Thus

$$4 + 16 + 36 = 56.$$

The resulting geometric contact graph contains

$$| E | = 252$$

total nearest-neighbor contacts, distributed by shell class as

$$E_{11} = 6, E_{12} = 36, E_{22} = 30, E_{23} = 84, E_{33} = 96.$$

4.3 Productive corridor optimization

Two nucleons are considered in contact when their lattice separation equals the nearest-neighbor spacing of the close-packed tetrahedral lattice. Only proton–neutron contacts produce productive interaction corridors. Optimizing the proton–neutron labeling of the 56 nodes under the constraint

$$Z = 26, N = 30$$

yields the productive corridor counts

$$N_{pn}^{(11)} = 4, N_{pn}^{(12)} = 24, N_{pn}^{(22)} = 20, N_{pn}^{(23)} = 56, N_{pn}^{(33)} = 64$$

so that the total productive corridor count of the closed Layer-3 cluster is

$$N_{pn}(56) = N_{pn}^{(11)} + N_{pn}^{(12)} + N_{pn}^{(22)} + N_{pn}^{(23)} + N_{pn}^{(33)}$$

$$N_{pn}(56) = 4 + 24 + 20 + 56 + 64 = 168$$

productive proton–neutron interaction corridors.

Thus the complete productive corridor decomposition of the 56-node Layer-3 cluster is

$$(4, 24, 20, 56, 64)$$

for the contact classes

$$(11, 12, 22, 23, 33).$$

4.4 Layer-3 spacing classes

The Layer-3 cluster contains five nearest-neighbor spacing classes

$$d_{11}, d_{12}, d_{22}, d_{23}, d_{33}.$$

From the Layer-1 and Layer-2 metric derivations,

$$d_{11} = d_{22} = d_{33} = d_{NN}$$

The Layer-3 shell compression derived from the general closed-shell rule gives

$$\delta_3 = Z_t \frac{\Delta C_3}{C_{\text{tot}}(3)} = 43 \times \frac{36}{56} = 27.642857\, L_0$$

Thus the mixed Layer-2 ↔ Layer-3 spacing is

$$d_{23} = d_{NN} - \delta_3 = d_{NN} - 27.642857 = 993.747543\, L_0$$

Therefore the Layer-3 spacing set is

$$d_{23} = 993.747543\, L_0, \qquad d_{33} = d_{NN}$$

Using the productive corridor weights derived in Section 4.3

$$(4,\ 24,\ 20,\ 56,\ 64)$$

the productive-corridor weighted spacing is

$$d_{\text{avg}} = \frac{4d_{11} + 24d_{12} + 20d_{22} + 56d_{23} + 64d_{33}}{168} = 1012.885714\, L_0$$

Using the full geometric contact graph derived in Appendix C

$$(E_{11}, E_{12}, E_{22}, E_{23}, E_{33}) = (6,36,30,84,96)$$

the contact-weighted effective spacing is

$$d_{\text{eff}} = \frac{6d_{11} + 36d_{12} + 30d_{22} + 84d_{23} + 96d_{33}}{252} = 1012.757143\, L_0 = 1.821174 \text{ fm}$$

Appendix A – Derivation of Cap/Exposure Coherence Function

A.1 Geometric Origin of the Corridor Exposure Angle

Consider two nucleons whose Zemach envelopes are modeled as equal spheres of radius r_Z separated by a center distance d. The intersection of the two Zemach spheres forms a circular lens region. The half-angle from the center of either sphere to the intersection circle defines the **corridor exposure angle** $\alpha(d)$.

From elementary sphere geometry,

$$\cos\ \alpha(d) = \frac{d}{2r_Z}$$

so that

$$\alpha(d) = \arccos\left(\frac{d}{2r_Z}\right)$$

This angle measures the portion of the Zemach shell that is geometrically available to participate in the nuclear interaction corridor. When $d \to 2r_Z$, the shells just touch and $\alpha \to 0$. As the separation decreases, the available interaction cap grows.

A.2 Corridor Sector Quantization

Within the EOTU framework, corridor interactions are restricted to discrete geometric sectors defined by the combined symmetry of the **four principal corridor directions** in the nucleon packing geometry the **threefold rotational structure of the interacting triads** Together these symmetries partition a full rotation into

$$4 \times 3 = 12$$

equivalent orientation sectors.

The fundamental corridor participation angle is therefore

$$\theta_0 = \frac{2\pi}{12} = \frac{\pi}{6}$$

This defines the angular unit over which a corridor-aligned triad interaction can occur.

A.3 Fractional Corridor Exposure

The fraction of an available corridor sector exposed by the Zemach overlap is therefore

$$F(d) = \min\left(1,\ \frac{\alpha(d)}{\theta_0}\right)$$

This dimensionless quantity measures the **degree of corridor alignment permitted by the geometric overlap of the Zemach shells**. When the overlap cap exceeds a full sector, the corridor exposure saturates.

A.4 Triad Coherence Projection

The interacting nucleons contain triad curvature structures whose active coupling mode is shared between the corridor axis and the intrinsic triad closure axis. In the symmetric interaction configuration these two directions form orthogonal basis components with equal weight.

The resulting projection factor is therefore

$$\cos\ \Delta,\Delta = \frac{\pi}{4}$$

so that

$$\cos\ \Delta = \cos\ \left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

This projection expresses the geometric alignment between the corridor exchange direction and the intrinsic triad curvature orientation.

A.5 Cap/Exposure Coherence Function

The effective coherence enhancement produced by the exposed corridor sector is obtained by combining

- the baseline interaction level
- two symmetric triad contributions
- the geometric projection factor
- the fractional corridor exposure

This yields the corridor coherence function

$$g_{pn}(d) = 1 + 2\cos\ \left(\frac{\pi}{4}\right) F(d)$$

and the coherence increment

$$\Delta g_{pn}(d) = g_{pn}(d) - 1$$

A.6 Resulting Corridor Binding Functional

Substituting the coherence factor into the corridor overlap interaction gives the universal proton–neutron corridor binding expression

$$E_{pn}(d) = \Gamma\left(\frac{8}{51}\right) V_{shell}(d)\Delta g_{pn}(d)$$

where

- $V_{shell}(d)$ is the engaged Zemach-shell overlap volume
- $8/51$ is the baryonic CPP participation fraction
- Γ is the SI bridge energy mapping constant.

This completes the geometric derivation of the coherence term used in the nuclear binding functional.

Appendix B- compression Rule Derivations

B.1 Layer 2 compression

When the outer 16-nucleon shell closes around the tetrahedral core, the closed corridor network slightly compresses the core–shell spacing.

The inward penetration determines the **geometric contact spacing** within the nuclear packing structure. This derivation yields the mixed core–shell spacing $d_{12}$and two useful cluster metrics: the productive-corridor weighted spacing and the geometric contact-graph effective spacing.

Productive-corridor weighted spacing

$$d_{avg} = \frac{4d_{11} + 24d_{12} + 20d_{22}}{48}$$

Contact-graph effective spacing

$$d_{eff} = \frac{E_{11}d_{11} + E_{12}d_{12} + E_{22}d_{22}}{E_{11} + E_{12} + E_{22}}$$

The simplest configuration that preserves the solved packing geometry keeps both the tetrahedral core and the shell lattice rigid.

$$\delta_2 = Z_t \frac{\Delta C_2}{C_{\text{tot}}(2)} = 43 \times \frac{16}{20} = 34.4\, L_0$$

so that

$$d_{12} = d_{NN} - \delta_2 = d_{NN} - 34.4 = 986.9904\, L_0$$

$$d_{22} = d_{11}, \qquad d_{12} = 986.9904\ L_0.$$

The Layer-2 closed shell corresponds to the tetrahedral cluster T_4consisting of lattice sites satisfying

$$i + j + k \leq 3, i, j, k \geq 0$$

on the tetrahedral close-packing lattice whose basis vectors are defined in Appendix C. This cluster contains 20 nodes and produces the contact graph

$$(E_{11}, E_{12}, E_{22}) = (6{,}36{,}30),$$

with $|E| = 72$ total contacts.

B.2 Layer 3 compression

The closed-shell $L = 3$ nucleus contains

$$C_{\text{tot}}(3) = 56$$

nucleons, with the outer shell population

$$\Delta C_3 = 36.$$

From the shell compression rule,

$$\delta_L = Z_t \frac{\Delta C_L}{C_{\text{tot}}(L)}, \qquad Z_t = 43\, L_0,$$

the Layer-3 compression is

$$\delta_3 = 43 \frac{36}{56} = 27.642857\, L_0.$$

The corresponding mixed Layer-2 ↔ Layer-3 nearest-neighbor spacing becomes

$$d_{23} = d_{NN} - \delta_3 = d_{NN} - 27.642857 = 993.747543\, L_0$$

Appendix C — Layer-3 Closed-Shell Geometric Construction

C.1 Purpose This appendix derives the closed-shell geometric packing for Layer-3 nuclei.

The derivation produces the complete **56-nucleon tetrahedral close-packed cluster**, which serves as the **master contact graph for all Layer-3 nuclei**. All later Layer-3 corridor calculations operate on induced subgraphs of this structure. The derivation is purely geometric and does not depend on proton–neutron assignment or the binding functional.

C.2 Tetrahedral Close-Packing Lattice

The tetrahedral lattice used here is the universal nuclear packing lattice.

Nucleon centers are placed on the tetrahedral close-packing lattice defined by the basis vectors

$$\mathbf{e}_1 = (1,0,0)$$

$$\mathbf{e}_2 = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right)$$

$$\mathbf{e}_3 = \left(\frac{1}{2}, \frac{\sqrt{3}}{6}, \sqrt{\frac{2}{3}}\right)$$

Any lattice site is

$$\mathbf{r} = i\mathbf{e}_1 + j\mathbf{e}_2 + k\mathbf{e}_3$$

where

$$i, j, k \in \mathbb{Z}.$$

The nearest-neighbor distance of this lattice is normalized to

$$| \mathbf{r}_i - \mathbf{r}_j | = 1.$$

Two nucleons are in geometric contact when this condition is satisfied.

C.3 Closed-Shell Tetrahedral Cluster

The Layer-3 closed shell corresponds to a **tetrahedral cluster of edge length**

$$n = 6.$$

The number of lattice sites inside such a tetrahedron is the tetrahedral number

$$T(n) = \frac{n(n+1)(n+2)}{6}.$$

For $n = 6$

$$T(6) = 56.$$

Thus the Layer-3 closed shell contains

nucleons.

C.4 Integer Lattice Coordinates

The 56 lattice sites are generated by the constraint

$$i + j + k \leq 5$$

with

$$i, j, k \geq 0.$$

This produces the complete tetrahedral cluster. The sites appear in six horizontal layers.

k = 0 21 sites, **k = 1** 15 sites, **k = 2** 10 sites, **k = 3** 6 sites, **k = 4** 3 sites, **k = 5** 1 site

Total $21 + 15 + 10 + 6 + 3 + 1 = 56$. Each integer triple $(i, j, k)$is converted to a physical coordinate using

$$\mathbf{r} = i\mathbf{e}_1 + j\mathbf{e}_2 + k\mathbf{e}_3.$$

C.5 Contact Definition

Two nucleons are considered in contact when

$$| \mathbf{r}_i - \mathbf{r}_j | = 1.$$

Applying this condition to the 56 sphere centers produces the **geometric contact graph** of the closed shell.

C.6 Resulting Contact Graph

The resulting edge counts separate naturally by shell class:

Contact class	**Edge count**
Layer-1 / Layer-1	6
Layer-1 / Layer-2	36
Layer-2 / Layer-2	30
Layer-2 / Layer-3	84
Layer-3 / Layer-3	96

Total edges

$$| E | = 252.$$

C.7 Role in the Layer-3 Model

The 56-node cluster derived above is the **master geometric structure** for Layer-3 nuclei.

All nuclei with

$$21 \leq A \leq 56$$

are treated as **induced subgraphs** of this cluster. Subsequent derivations determine

- proton–neutron assignments,
- productive corridor counts,
- and effective spacing metrics

on this fixed geometric foundation.

Appendix D – α-cluster backbone ladder

$$E_{\text{bind}} = N_\alpha E_\alpha + n_4 E_{d4} + n_5 E_{d5} + n_6 E_{d6},$$

$$E_\alpha = 28.2959 \text{ MeV}$$

Nucleus	**α clusters**	$N_\alpha E_\alpha$**(MeV)**	**Nucleus**	**α clusters**	$N_\alpha E_\alpha$**(MeV)**	**Nucleus**	**α clusters**	$N_\alpha E_\alpha$**(MeV)**
^{4}He	1 α	28.2959	^{28}Si	7 α	198.0713	^{52}Fe	13 α	367.8467
^{8}Be	2 α	56.5918	^{32}S	8 α	226.3672	^{56}Ni	14 α	396.1426
^{12}C	3 α+	92.1112	^{36}Ar	9 α	254.6631	^{60}Zn	15 α	424.4385
^{16}O	4 α	113.1836	^{40}Ca	10 α	282.9590	^{64}Ge	16 α	452.7344
^{20}Ne	5 α	141.4795	^{44}Ti	11 α	311.2549	^{68}Se	17 α	481.0303
^{24}Mg	6 α	169.7754	^{48}Cr	12 α	339.5508	^{72}Kr	18 α	509.3262

The milestone anchors occur at $A = 4N_\alpha$. Each anchor adds one α cluster to the preceding closure while preserving the same standard corridor set (d_4, d_5, d_6). The resulting binding energies indicate that the dominant structural change is not a new interaction scale, but a re-occupation of the same corridor distances as the α-cluster backbone grows.

By ^{32}S, and ^{36}Ar, multiple corridor combinations can approach the same measured energy, suggesting that higher Layer-3 closures admit more than one local packing arrangement within the same shell family. At higher mass **nuclei**, multiple corridor combinations can approach the measured energy. Therefore the structural interpretation must also consider geometric closure of the alpha cluster network.

Direct coordinate-graph testing shows that the Ca-40 clustered geometry extends into Ti-44 and Cr-48 through three discrete tetrahedral cap sites remaining on the Layer-3 master cluster. Ti-44 corresponds to occupation of any one of these symmetry-equivalent caps, while Cr-48 corresponds to occupation of any two. Thus both nuclei admit clean clustered geometric realizations, and the post-Ca sequence appears to proceed by discrete cap completion rather than arbitrary rearrangement.

Geometric constraint on corridor partition.
Direct testing of the ^{40}Ca configuration shows that while the total corridor count is preserved, the finer d_5/d_6 decomposition is constrained by the discrete interface structure of the α-cluster network. In particular, the strong-interface contribution is quantized in even increments, making the energy-optimal $(17, 7, 25)$ partition geometrically inadmissible. The nearest valid realization $(17, 8, 24)$ satisfies all geometric constraints, introducing a small but systematic deviation from the energy minimum. This indicates that at higher A, geometric closure governs the fine structure of the corridor decomposition.

Appendix E – Derivation for Delta δ for the shell

1. The neutron contributes a small excess curvature load

The neutron is slightly heavier than the proton, so in a coupled p-n system its shell is not perfectly symmetric with the proton shell. In your model that should appear not as a full geometric resize, but as a **small outward bias** in the effective contact radius.

Represent that asymmetry by the dimensionless ratio

$$\varepsilon_{np} = \frac{\Delta m_{np}}{m_p}$$

2. The mediator doubles the proton-facing response

In your pictured geometry, the electron sits in the coupled corridor between the two proton-facing shells. That means the asymmetry is not acting on only one side of one nucleon; it is being felt through a **two-sided coupled curvature channel**. That is why a factor of **2** is natural: pair response $\sim 2\varepsilon_{np}$

3. Only the corridor-active part of the Zemach shell can relax

You already defined that of the 51-cell shell, 8 cells are baryonic-active and the rest are curvature-active. Across the full double-shell thickness $102\,L_0$, the relaxable fraction is

$$\frac{102-8}{102} = \frac{94}{102} = \frac{47}{51}$$

So the neutron excess does **not** push the whole shell equally. It only pushes the corridor-relaxing part:

$$\text{screening factor} = \frac{47}{51}$$

4. Apply that perturbation to the locked pair radius

Once the coupled system exists, the natural radius to perturb is the binary lock radius itself:

$$r_{\text{lock}} = 512\,L_0$$

So the induced correction is

$$\delta_z \approx r_{\text{lock}}\left(2\frac{\Delta m_{np}}{m_p}\right)\left(\frac{47}{51}\right)$$

Numerically this gives $1.3008\,L_0$

Interpretation That gives you a very usable statement:

δ_z is the mediator-screened neutron excess projected into the corridor-active Zemach shell.

Appendix W: minimal overlap energy-maximum spacing

$$E_{pn}(d) = \Gamma\left(\frac{8}{51}\right) V_{\text{shell}}(d)\, \Delta g_{pn}(d)$$

with:

- $r_Z = 569.6952\, L_0$
- $t = 43\, L_0$
- $\theta_0 = \pi/6$
- $\Gamma = 1$ eV per L_0^3

2) Geometry at $d = 1136\, L_0$ Overlap condition

$$2r_Z = 1139.3904\, L_0$$

$$2r_Z - d = 3.3904\, L_0 \text{(very small overlap)}$$

3) Coherence term

$$\alpha(d) = \arccos\left(\frac{d}{2r_Z}\right) = \arccos(0.99702) \approx 0.077 \text{ rad}$$

$$F(d) = \frac{\alpha}{\theta_0} \approx \frac{0.077}{0.5236} \approx 0.147$$

$$g_{pn} = 1 + 2\cos\left(\frac{\pi}{4}\right) F = 1 + 2(0.7071)(0.147) \approx 1.2077$$

$$\Delta g_{pn} \approx 0.2077$$

4) Shell overlap volume Outer lens:

$$V_{\text{lens}}(r_Z, d) = \frac{\pi(4r_Z + d)(2r_Z - d)^2}{12}$$

$$(2r_Z - d) = 3.3904, (2r_Z - d)^2 \approx 11.5$$

$$4r_Z + d = 3414.7808$$

$$V_{\text{lens}} \approx 1.03 \times 10^4\, L_0^3$$

Inner radius:

$$r_Z - t = 526.6952 \Rightarrow 2r < d \Rightarrow V_{\text{inner}} = 0$$

$$V_{\text{shell}} \approx 1.03 \times 10^4\, L_0^3$$

5) Energy

$$E_{pn}(1136) = 1 \cdot \left(\frac{8}{51}\right) \cdot 10287 \cdot 0.2077$$

$$\frac{8}{51} \approx 0.15686$$

$$E \approx 335 \text{ eV}$$

Appendix X: Fixed node numbering for the 56-node cluster

Define the closed-shell Layer-3 cluster as the integer tetrahedral set

$$\mathcal{T}_5 = \{(i,j,k) \in \mathbb{Z}^3_{\geq 0} : i + j + k \leq 5\}.$$

Number nodes in this exact order:

1. increasing k,
2. within each k, increasing j,
3. within each (j, k), increasing i.

Equivalently: scan each horizontal layer from bottom to top, row by row, left to right.

Node table

$k = 0$ layer

1 : (0,0,0)
2 : (1,0,0)
3 : (2,0,0)
4 : (3,0,0)
5 : (4,0,0)
6 : (5,0,0)
7 : (0,1,0)
8 : (1,1,0)
9 : (2,1,0)
10 : (3,1,0)
11 : (4,1,0)
12 : (0,2,0)
13 : (1,2,0)
14 : (2,2,0)
15 : (3,2,0)
16 : (0,3,0)
17 : (1,3,0)
18 : (2,3,0)
19 : (0,4,0)
20 : (1,4,0)
21 : (0,5,0)

$k = 1$ layer

22 : (0,0,1)
23 : (1,0,1)
24 : (2,0,1)
25 : (3,0,1)
26 : (4,0,1)
27 : (0,1,1)
28 : (1,1,1)
29 : (2,1,1)
30 : (3,1,1)
31 : (0,2,1)
32 : (1,2,1)
33 : (2,2,1)
34 : (0,3,1)
35 : (1,3,1)
36 : (0,4,1)

$k = 2$ layer

37 : (0,0,2)
38 : (1,0,2)
39 : (2,0,2)
40 : (3,0,2)
41 : (0,1,2)
42 : (1,1,2)
43 : (2,1,2)
44 : (0,2,2)
45 : (1,2,2)
46 : (0,3,2)

$k = 3$ layer

47 : (0,0,3)
48 : (1,0,3)
49 : (2,0,3)
50 : (0,1,3)
51 : (1,1,3)
52 : (0,2,3)

$k = 4$ layer

53 : (0,0,4)
54 : (1,0,4)
55 : (0,1,4)

$k = 5$ layer

56&:(0,0,5)

Shell grouping under this numbering

This numbering is geometric, not shell-labeled. So for Section 4, you should define shell membership separately.

A clean canonical shell split is:

- **Layer 1:** nodes with $i + j + k \leq 1$, so

$$\{1, 2, 7, 22\}$$

- **Layer 2:** nodes with $i + j + k = 2$ or 3, total 16 nodes
- **Layer 3:** nodes with $i + j + k = 4$ or 5, total 36 nodes

That gives the required

$$4 + 16 + 36 = 56.$$

Explicit shell sets

Layer 1

$$\{1, 2, 7, 22\}$$

Layer 2

$$\{3, 8, 23, 12, 13, 27, 37, 4, 9, 24, 14, 28, 38, 16, 31, 41\}$$

Layer 3

{5, 10, 25, 15, 29, 39, 17, 32, 42, 19, 44, 47, 6, 11, 26, 18, 33 , 43, 20, 35, 45, 21, 36, 46, 48, 30, 40, 34, 49, 50, 53, 54, 51, 55, 52, 56}

Rule for future threads

When referencing any node:

- always cite its ID first,
- then optionally its lattice coordinate (i, j, k).

Example:

- "node 43 $(2, 1, 2)$"
- "edge $(29, 33)$"
- "occupied Layer-3 subset includes nodes 48, 50, 53"

One-line document rule

You can lock it with this sentence:

The 56-node Layer-3 master cluster is numbered canonically by tetrahedral lattice coordinates $(i, j, k)$with $i + j + k \leq 5$, ordered by increasing k, then increasing j, then increasing i.

Appendix Y – Layer-1 Geometric Derivation

Derivation Context Only- Not used for direct binding evaluation

For $L = 1$the lattice constraint $i + j + k \leq 2L - 1$ produces four lattice sites, forming the regular tetrahedral nucleus.

Using the previously defined corridor functional

$$E_{pn}(d) = \Gamma\left(\frac{8}{51}\right) V_{\text{shell}}(d)\, \Delta g_{pn}(d)$$

the total nuclear binding energy is

$$E_{\text{bind}} = \sum_{(i,j)\in\mathcal{L}_{pn}} E_{pn}\,(d_{ij})$$

where $\mathcal{L}_{pn}$ is the set of proton–neutron corridors. A proton–neutron corridor exists when two nucleons satisfy Corridor condition

$$d_{ij} \leq 2r_{nuc}$$

Corridor set

$$\boxed{\mathcal{L}_{pn}(A,Z) = \{(i,j) \in E(A) \mid \ell(i) = p, \ell(j) = n, d_{ij} \leq 2r_{nuc}\}}$$

where

- $E(A)$= nearest-neighbor edges of the nucleon packing graph
- $\ell(i)$= nucleon label (p or n)

Only proton–neutron pairs form binding corridors. Proton–proton and neutron–neutron pairs do not produce a binding corridor.

Y.1 Core Packing

The p-n are tetrahedral packed to form layers

$$r_{core}(Z) = L_{\text{core}}(Z)\, r_{\text{pn}} = L_{\text{core}}(Z)\, [r_{\text{mass}} + (51 - 8)\, L_0]$$

interaction boundary where r_{pn} = This boundary lies inside the Zemach radius and outside the mass boundary. So a nucleus is: A cluster of r_{pn}-sized spheres packed as tightly and symmetrically as possible. These structures arise from geometric packing of equal-radius nucleon regions constrained by the layer capacities $\Delta C_L = 4L^2$.

- Each p & n is a 3-D composite CPP region,
- Each p & n occupies a rigid spherical volume of radius defined by the Zemach radius

- Any overlap is at Zemach shell
- p-n form preferred pairs whenever possible
- p-p avoid each other if possible
- If p-p adjacency, curvature and distance is increased (must maintain curvature-stable symmetry with neighbors)
- n-n contact is allowed but not preferred
- Extra neutrons are on the outer shell only
- The p–p and n–n contacts are geometric/nonbinding contacts, with p–p naturally interpreted as curvature-defect sites rather than corridor channels.

All nuclear packing layers arise as truncated clusters of the tetrahedral close-packing lattice

$$\mathbf{r} = i\mathbf{e}_1 + j\mathbf{e}_2 + k\mathbf{e}_3$$

with

$$i, j, k \geq 0$$

and the shell depth constraint

$$i + j + k \leq 2L - 1$$

where $L$is the nuclear packing layer.

Y.2 Layer 1 packing

- Layer 1 is a tetrahedral 4-nucleon shell.
- It has 6 geometric contacts.
- Of the 6 geometric contacts, 4 are productive p–n corridors.

Y.3 Layer-1 Tetrahedral Core Derivation (A = 4)

Layer-1 forms the seed structure of the nucleus packing geometry.

The nucleus is modeled as equal-radius nucleon interaction spheres with contact distance

$$d_{NN} = 2r_{nuc}, \qquad r_{pn} = r_{mass} + Z_t$$

Layer-1 contains four nucleons arranged in the maximally symmetric configuration of four equal spheres, which is a regular tetrahedron.

$$d_{11} = Z_t$$

Tetrahedral geometry: A tetrahedron with four vertices has

$$\binom{4}{2} = 6$$

pairwise edges. Each edge corresponds to a geometric contact between nucleons. Therefore, the Layer-1 packing has $E_{11} = 6$ geometric contacts.

Proton–neutron labeling: For the balanced configuration

$$A = 4, \qquad Z = 2, \qquad N = 2$$

the optimal assignment maximizes p–n neighbors. The tetrahedron is a complete graph K_4. With two protons and two neutrons the contact distribution becomes 4 p–n contacts, 1 p–p contact, 1 n–n contact

Productive binding corridors: Only p–n contacts produce productive corridors. Therefore, the number of productive binding corridors is

$$N_{pn}(4,2) = 4$$

Result Layer-1 **internal contacts** form a **tetrahedral four-nucleon seed structure** with

$$E_{11} = 6$$

geometric contacts and

$$N_{pn} = 4$$

productive p–n binding corridors. This structure corresponds to the **He-4 closed tetrahedral core** that seeds all higher shells.

$$d_{11} = 2r_{nuc}$$

Appendix Z- Nucleus Allowed and Disallowed States

Nuclear binding arises from corridor-mediated curvature minimization between nucleons as described by the proton–neutron corridor binding functional in §1.

Z.1 Fundamental Nucleon Structures

Within the EOTU framework:

Proton

Stable Region composed of a closed **UUDZZZ curvature core**.

Neutron

Composite Region consisting of

$$n = (UUDZZZ\ core + EZZZ\ shell)$$

The neutron is therefore a **proton structure with an encapsulated electron shell** and is metastable in isolation but stabilizes inside a nuclear Region.

Z.2 Nuclear Pairing Primitive

The smallest stable nuclear coupling is the **proton–neutron dimer**

$$(p \parallel n)$$

This configuration forms when two nucleons meet the nuclear corridor condition and their Zemach shells overlap. The pairing mechanism arises because:

- the proton provides the stable curvature core
- the neutron contributes an internal EZZZ shell
- the interaction occurs within the Zemach boundary layer

The result is a stable **nuclear interaction corridor** between the two nucleons.

Z.3 Absence of Pure Proton Nuclei

Beyond the single-proton nucleus (protium), aggregates composed only of protons do not form stable nuclear Regions. Within the EOTU this occurs because:

- proton structures contain only **UUDZZZ closure shells**
- coupling would therefore occur through **Z-closure overlap alone**
- this configuration does not provide sufficient phase/curvature structure to maintain a stable nuclear corridor.

Thus **pure proton aggregates cannot stabilize** beyond $Z = 1$.

Z.4 Absence of Pure Neutron Nuclei

A neutron contains an **encapsulated electron shell (EZZZ)**. When neutrons attempt to couple directly:

- the nuclear corridor becomes dominated by **EZZZ–EZZZ shell interaction**
- the metastable shell component lacks a stabilizing proton curvature boundary

Therefore neutron–neutron aggregates do not form stable nuclear Regions.

Z.5 Mixed Nucleon Requirement

Stable nuclei therefore require **mixed proton–neutron pairing**. The proton provides structural curvature closure, while the neutron provides an additional internal shell degree of freedom that can participate in corridor formation without forcing mass-shell overlap. The fundamental nuclear interaction unit is therefore the **p–n pair**.

Z.6 Neutron Excess in Larger Nuclei

As the number of nucleons increases, the number of required nuclear corridors increases. Additional protons increase curvature density within the nucleus and therefore increase the number of stabilization channels required. Neutrons supply these channels because their encapsulated EZZZ shell introduces additional coupling degrees of freedom without adding a new exposed charge channel.

Consequently stable nuclei progressively require

$$N > Z$$

for larger nucleon counts.

Z.7 Why Pure Proton Nuclei Do Not Exist

In the EOTU framework nuclear binding arises from **proton–neutron corridor coupling** governed by the triad coherence rule.

The binding energy of a corridor link is

$$E_{pn}(d) = \Gamma\left(\frac{8}{51}\right) V_{\text{shell}}(d)\, \Delta g_{pn}(d)$$

where the coherence term

$$\Delta g_{pn}(d) = g_{pn}(d) - 1$$

is determined by the allowed triad phase channels.

Triad Coherence Requirement

The stable corridor channel requires the triad phase configuration

two half-phase edges

$$\Delta\phi = \frac{\pi}{2}$$

one aligned edge

$$\Delta\phi = 0$$

This asymmetric configuration allows constructive corridor coherence between nucleons.

Proton–Neutron Coupling

A proton–neutron pair naturally satisfies this configuration because the exposed curvature patches on the proton and neutron carry **complementary triad phases**.

This produces a positive coherence excess

$$\Delta g_{pn}(d) > 0$$

which yields a non-zero binding energy.

Proton–Proton Interaction

Two protons present identical triad phase structures.

Because their exposed curvature patches are phase-aligned, the required mixed phase configuration

$$(\pi/2, \pi/2, 0)$$

cannot be constructed.

As a result the coherence factor becomes

$$g_{pp}(d) \approx 1$$

and therefore

$$\Delta g_{pp}(d) = 0$$

which gives

$$E_{pp}(d) = 0$$

Consequence

Although proton–proton corridors may exist geometrically, they cannot form a stable triad coupling channel and therefore do not produce nuclear binding.

As a result:

Pure proton nuclei cannot form bound states

A neutron is required to provide the complementary phase structure that enables corridor coherence and nuclear binding.

Physical Interpretation

Within this model the neutron functions as a **phase mediator** that allows stable corridor engagement between nucleons.

This explains why:

- the **deuteron requires a neutron**
- stable nuclei contain **both protons and neutrons**
- proton–proton systems do not form bound nuclei.